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MENTAL ARITHMETIC.

BY

CHARLES G. BURNHAM, A. M.

BEING

AN INTRODUCTION

TO HIS

NEW SYSTEM OF ARITHMETIC ON AN IMPROVED PLAN.

BOSTON:
JOHN P. JEWETT AND COMPANY.
CLEVELAND, O.:
HENRY P. B. JEWETT.
1859.

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PREFACE.

THIS book, Part I, a work on Mental Arithmetic, is designed to be introductory to Part II, a work on Written Arithmetic, in which the science of Arithmetic is more fully developed, and the system rendered more complete. Part I. differs in many respects from any work of the kind now in use. It is peculiar in its commencement and mode of procedure. It commences where the child commences, and proceeds as the child proceeds, —falls in with his own mode of arriving at truth,—discovers to him his process,—cultivates his desire to know the *cause*, and furnishes him with language properly to express his thoughts. In this way we aid the child to think for himself, excite his interest, and secure his future progress. Hence a series of introductory exercises are given, by which the child is made familiar with his own mental operations,—the mystery of his own thoughts is revealed, and he learns to express in the language of mathematical demonstration, those truths which have ordinarily been charged over to memory, or been considered intuitive. But where does the child commence, and how does he proceed? What is the natural order in which his thoughts are suggested? Obviously, his first idea is of unity. An object passes before his eye, and the idea of a *thing* is suggested. Another object passes, and the idea of succession, of *another* thing, follows. Contrast and comparison succeed, and the idea of individuality (or the distinctive traits of character peculiar to each) is the result. With the foregoing exercises and thoughts, the mind of the child is occupied until the power of language is acquired, when it learns to count and apply names to the succession of things which passes before it. A single formula is given in this work, by which

the mind, through the eye, may be aided in comprehending those few elementary truths, which are the foundation of all mathematical calculations. Only one is given, because by it all those truths may be illustrated; and when they are clearly perceived, no more objects should be presented to the eye. Since the study of mathematics is eminently calculated to improve the reflective faculties, it is important to remove from the sight every object that would interrupt or lessen the labor of reflection. Having illustrated by the formula a few simple truths, remove it from before the pupil, and teach him to reflect upon these truths, and show him how to make use of the knowledge he has already gained, in acquiring more. A familiarity with the simple and earliest processes of the mind in search of truth, will not be deemed unimportant by those who require the same of the more advanced pupil in all analytic and synthetic processes. Still there will be objectors to the mode here adopted. Some will object to its novelty, some to its simplicity, and others to its intricateness. The first objection has already been done away by the practices of the present age. To the second it may be replied, that it is nature's own mode, and therefore it is the best; and this sufficiently meets the third objection. But it may be added, that if it be found difficult, this fact indicates that the child is not sufficiently advanced to commence the study of Mental Arithmetic; for no child should commence the study of any science, until he is able to comprehend its simplest elements.

But at a much earlier age than many are aware, children employ more difficult mental processes than are necessary in the commencement of the study of the science of numbers. Even when they can command but few words, and arrange these but imperfectly in sentential order, their reasonings give birth to feelings and sentiments, most of which it is true, like early fruit, quickly perish, but some, indeed, live, and are fresh even in life's winter; and not unfrequently, too, is the eye of childhood moistened with tears, and the heart saddened by the workings of the mind within. A single instance will serve to illustrate. The

younger brother of a little boy, two and a half years of age, sickened and died. The mother was also sick. One day by himself alone, the little boy was seen to burst into tears, and to weep bitterly. Being asked why he wept, he replied, Mother will die. He was asked why he thought his mother would die. He replied, Brother was sick, and died: mother is sick, and she will die. The experience of the child fully established this proposition. He had observed sickness to terminate in death, and had never known it otherwise. His reasoning was logical; it was natural that he should rely upon his conclusion with implicit confidence. He had taken into consideration the observation of his whole life, and a man of threescore years and ten could do no more. But he failed in one point; he did not know that a single fact was not enough to establish a principle. This child was not remarkable for early mental development. The case is a fair sample of the reasonings of children at an early age.

The importance of adapting our teachings to the laws of mind,—of seeking to excite thoughts in their natural order,—of making every new idea the result of a previous one, is too obvious to need discussion.

TO TEACHERS.

Teachers will be careful to adopt and carry out the mode of instructing here proposed, unless they can substitute a better. In all questions involving analysis and synthesis, the process of the solution should be insisted on.

The teacher can adopt his own mode of recitation, whether to read the question himself, or require the pupil to read it. In all cases, he should require the pupil to be so familiar with his lesson, as to be able to recite rapidly and correctly. It is no unimportant part of education to be able to bring our mental powers to act instantly and efficiently upon a given subject.

Throughout the work, after each question, the process is generally required. This is to remind both teacher and scholar of

the necessity of always giving it. When the question is proposed, the scholar should answer it, and give the process immediately, without being called upon so to do.

The extent to which the question "How do you know?" should be repeated, must be left to the discretion of the teacher. It should, in all cases, be repeated until the pupil clearly perceives and is familiar with his reasons. A neglect properly to train the mind to systematic thinking, has operated, probably, more than any other one cause to enfeeble the mental energies of our whole race.

Teachers, would you secure the improvement of your pupils? seek to improve yourselves. Be students in mental philosophy. Nor can you avail yourselves of a more favorable opportunity to pursue the study of this most interesting science, than by devoting a portion of your time to the instruction of the young. Those innumerable questions of children, which to others are unmeaning, will to you be full of interest. They are the effect of a cause, and that cause you will seek to know. They are questions which the wisest of men would ask, ignorant of the same facts. The little boy who says he is not five years old, but knows that he is six, would be as much pleased as any one else to hear the remark made by another, did he himself know the order in which the names of numbers are applied to things. He does not know that he cannot be six until he is five; for he has not learned but that he can commence at the other end of the series, in which case he would come to six first. Again: the little boy stretches himself a little above his ordinary height, and asks if he is not—six years old. This child evidently confounds age with height. Nor is it certain that his previous instruction has not led him into this mistake. The father had, undoubtedly, told the child, that when he was six years old, "a big boy," elevating his hand at the same time to a certain height, that he would make him some present: thus virtually telling him, that height and age were synonymous terms. It is not unfrequently the case, that if you require the child, when he has learned to repeat the names of numbers, to count your fingers, that he will p

his finger on unity, or one, and count five: thus showing that he does not know that the names of numbers apply to distinct units placed in a certain order. And even when he has learned to count, he not unfrequently supposes that the number which he calls five, for example, would be five if standing alone, as John is the name of a boy. This mistake arises from his misapprehending the names of numbers for the names of things. Nor can the child always tell, even when he has learned to count, how many are one and one. This is a new mode of combining numbers; it contains a new idea, and is a thing to be learned.

The foregoing facts, and many more which might be mentioned, are well known to those familiar with children. Hence, when the teacher has written the formula upon the *blackboard*, he will direct the attention of the class to the following particulars:—

Teacher,—The class will notice these short lines, which I have drawn upon the *board*. We will call them units, or ones. They may be used to represent number, and this number may be of things of any kind, as chairs, tables. If I wished to inform you that I had two apples, I might do so by drawing two lines, which would represent their number; and these same lines might be used to represent two things of any other kind. These lines here show how many units the figures written under them stand for. We make use of the figures for the same purpose as we do the lines, to represent number. It is a shorter process to write a figure than to make as many lines as a single figure would represent.

You will also notice, that as you count, every new name applied, or unit counted, adds one unit to the preceding; this is one mode of adding. You will observe, also, that if I commence at the left hand and count five, and then count the same things back, from right to left, the names of the numbers are changed; the unit which I first called one, I now call five. The names, therefore, which we use in counting, are names of numbers, and not names of things; and it depends upon the place a thing occupies in relation to other things, what number it shall be called.

The number two comes after one, and three after two, etc. John, how many more than one is two? How many, then, are one and one? Then one from two, how many remain? You must remember what I have now told you, for it will aid you in preparing your next lesson.

The teacher will do well, at the commencement of each lesson, to review the leading thoughts of the previous lesson; for it is the design of this work to appropriate to immediate use every item of knowledge gained. Teachers, you will have need of patience and perseverance; your labor is arduous, and often your prospects of success are discouraging. But your calling is high,—it is noble. Your Great Exemplar was He by whom the worlds were made. Be encouraged, therefore; your reward is before you. Let the assistance which you render your pupils, be an aid to their own mental efforts, and not a substitute for them. Come down to the child, and lead him up higher; accompany him in his upward progress, and do not undertake to force him to come up alone. Employ suitable illustrations on subjects suited to his capacities. Let him behold truth in pleasing variety, robbed of none of its intrinsic beauty from a want of interest on your part. Feed him with knowledge; lead him daily by still waters and in green pastures.

MENTAL ARITHMETIC.

SECTION I.

Teacher. Well, my little scholars, you are about to commence a new study, and although it is the study of Arithmetic, yet you are not to make use of pencil and slate, as older scholars do, but you are to work out the questions in your mind.

George, do you know what I mean by working them in your mind?

S. In our thoughts.

T. Very well, and when you have worked them out in your thoughts, you will then express, to your teacher, those thoughts in words, and this will be a recitation.

James, when I say the class in mental arithmetic may recite, what will you understand me to mean?

S. That we may express to you, in words, our thoughts upon our lesson.

T. And what is the object of expressing to your teacher your thoughts upon your lesson?

S. That we may know whether our thoughts have been correct.

T. We will then proceed to our

FIRST LESSON.

Teacher. George, can you count?

Scholar. I can.

T. Let me hear you.

S. One, two, three, four, etc.

T. Do you know the names of figures?
S. I do.
T. What is the name of this figure or character, 1?
S. One.
T. What of this, 2?
S. Two.
T. What of these, 3, 4, 5?
S. Three, four, five?
T. What does the figure two stand for?
S. Two ones; as two chairs.
T. What does the figure three stand for?
S. Three ones.
T. What does the figure five stand for?
S. Five ones.
T. As you count from one to five, how many ones do add for each figure counted?
S. One.
T. Very well. I will illustrate this to you on the *bl board.* (See explanation of the formula.)

1.	11.	111.	1111.	11111.
1.	2.	3.	4.	5.
One.	Two.	Three.	Four.	Five.

LESSON SECOND.

Teacher. How many more than one is two?
Scholar. One.
T. How many more than two is three?
S. One.
T. How many more than one is three?
S. Two.
T. How do you know?
S. I know, because two is one more than one, and th is one more than two; therefore three is two more than o
T. How many more than three is four?
T. How many more than two is four?
S. Two.

T. How do you know?

S. Four is one more than three, and three is one more than two; therefore four is two more than two.

T. How many more than one is four?

T. Do you perceive any advantage in writing the characters 2, 3, 4, instead of so many ones?

S. It is a shorter mode of expression.

LESSON THIRD.

Teacher. How many are one and one?

Scholar. Two.

T. How do you know?

S. I know, because two is one more than one; therefore one and one are two.

T. How many are two and one?

S. Three.

T. How do you know?

S. Because three is one more than two; therefore two and one are three.

T. You say that two is one more than one, and three is one more than two; how many more than one is three?

T. How many are three and one?

S. Four.

T. How many are two and two?

S. Four.

T. How do you know?

S. Because four is one more than three, and two and one are three; therefore, two and two are four.

T. Are two and two the same as three and one?

S. They are.

T. How do you know?

S. Because three and one are four, and two and two are four; therefore they are the same.

T. How many more than four, is five?

S. One.

T. How many more than three, is five?

S. Two.

LESSON FOURTH.

T. How many are four and one?

S. Five.

T How do you know?

S. Because five is one more than four; therefore, four and one are five.

T. How many are three and two?

S. Five.

T. How do you know?

S. Because five is two more than three; therefore, three and two are five.

T. How many are two and three?

S. Five.

T. Are two and three the same as three and two?

S. They are.

T. How do you know?

S. Because they are the same numbers repeated in a reversed order.

T. What do you mean by reversed order?

S. If when we have said three and two are five, we immediately say, two and three are five, we reverse the order.

T. Three and three are how many?

S. Six.

T. How do you know?

S. Six is one more than five, and three and two are five, and three is one more than two; therefore, three and three are six.

LESSON FIFTH.

QUESTION.—Mary had one cent, and her mother gave her another, how many then had she? How many are one and one? How do you know?

T. What is the above process called?

S. Addition.

T. What, then, is addition?

S. It is putting together two or more numbers, so as to make but one.

T. For what purpose do you wish to put numbers together?

S. To find their sum, or amount.

T. For what purpose do you wish to find the amount in the above question?

S. To ascertain how many cents Mary had.

LESSON SIXTH.

QUEST. 1.—James had two cents, and his father gave him another; how many then had he?

Two and one are how many?

How do you know?

2.—Jane had two cents, and her father gave her two more; how many then had she?

Two and two are how many?

How do you know?

LESSON SEVENTH.

QUEST. 1.—Julia had three apples, and her companion gave her two more; how many then had she?

Three and two are how many?

How do you know?

Are two and three the same as three and two?

How do you know?

2.—Joseph had three apples, and his playmate gave him three more; how many then had he?

Six.

Three and three are how many?

How do you know?

3.—John had four peaches, and James gave him three more; how many then had he?
How many are four and three?
How do you know?

LESSON EIGHTH.

QUEST. 1.—Charles had five cents, and a friend gave him three more; how many then had he?
Five and three are how many?
How do you know?

2.—Thomas had five cents, and a friend gave him four more; how many then had he?
Five and four are how many?
How do you know?

3.—William had five apples, and Stephen gave him five more; how many then had he?
Five and five are how many?
How do you know?

LESSON NINTH.

QUEST. 1.—George had six dollars, and his uncle gave him five more; how many then had he?
Six and five are how many?
How do you know?

2.—Isaac had six doves; in time, six more were added to his flock; how many then had he?
Six and six are how many?
How do you know?

3.—Peter had seven cents, and he earned six more; how many then had he?
Seven and six are how many?
How do you know?

ADDITION TABLE.

A.					B.				
1	and	0	are	1	3	and	0	are	3
1	and	1	are	2	3	and	1	are	4
2	and	1	are	3	3	and	2	are	5
2	and	2	are	4	3	and	3	are	6
2	and	3	are	5	3	and	4	are	7
2	and	4	are	6	3	and	5	are	8
2	and	5	are	7	3	and	6	are	9
2	and	6	are	8	3	and	7	are	10
2	and	7	are	9	3	and	8	are	11
2	and	8	are	10	3	and	9	are	12
2	and	9	are	11	3	and	10	are	13
2	and	10	are	12	3	and	11	are	14

C.					D.				
4	and	0	are	4	5	and	0	are	5
4	and	1	are	5	5	and	1	are	6
4	and	2	are	6	5	and	2	are	7
4	and	3	are	7	5	and	3	are	8
4	and	4	are	8	5	and	4	are	9
4	and	5	are	9	5	and	5	are	10
4	and	6	are	10	5	and	6	are	11
4	and	7	are	11	5	and	7	are	12
4	and	8	are	12	5	and	8	are	13
4	and	9	are	13	5	and	9	are	14
4	and	10	are	14	5	and	10	are	15
4	and	11	are	15	5	and	11	are	16
4	and	12	are	16	5	and	12	are	17

E.					F.				
6	and	0	are	6	7	and	0	are	7
6	and	1	are	7	7	and	1	are	8
6	and	2	are	8	7	and	2	are	9
6	and	3	are	9	7	and	3	are	10
6	and	4	are	10	7	and	4	are	11
6	and	5	are	11	7	and	5	are	12

6	and	6	are	12
6	and	7	are	13
6	and	8	are	14
6	and	9	are	15
6	and	10	are	16
6	and	11	are	17
6	and	12	are	18

7	and	6	are	13
7	and	7	are	14
7	and	8	are	15
7	and	9	are	16
7	and	10	are	17
7	and	11	are	18
7	and	12	are	19

G.

8	and	0	are	8
8	and	1	are	9
8	and	2	are	10
8	and	3	are	11
8	and	4	are	12
8	and	5	are	13
8	and	6	are	14
8	and	7	are	15
8	and	8	are	16
8	and	9	are	17
8	and	10	are	18
8	and	11	are	19
8	and	12	are	20

H.

9	and	0	are	9
9	and	1	are	10
9	and	2	are	11
9	and	3	are	12
9	and	4	are	13
9	and	5	are	14
9	and	6	are	15
9	and	7	are	16
9	and	8	are	17
9	and	9	are	18
9	and	10	are	19
9	and	11	are	20
9	and	12	are	21

I.

10	and	0	are	10
10	and	1	are	11
10	and	2	are	12
10	and	3	are	13
10	and	4	are	14
10	and	5	are	15
10	and	6	are	16
10	and	7	are	17
10	and	8	are	18
10	and	9	are	19
10	and	10	are	20
10	and	11	are	21
10	and	12	are	22

J.

11	and	0	are	11
11	and	1	are	12
11	and	2	are	13
11	and	3	are	14
11	and	4	are	15
11	and	5	are	16
11	and	6	are	17
11	and	7	are	18
11	and	8	are	19
11	and	9	are	20
11	and	10	are	21
11	and	11	are	22
11	and	12	are	23

K.

12	and	0	are	12
12	and	1	are	13
12	and	2	are	14
12	and	3	are	15
12	and	4	are	16
12	and	5	are	17
12	and	6	are	18
12	and	7	are	19
12	and	8	are	20
12	and	9	are	21
12	and	10	are	22
12	and	11	are	23
12	and	12	are	24

L.

1	and	1	are	2
10	and	10	are	20
2	and	2	are	4
20	and	20	are	40
3	and	3	are	6
30	and	30	are	60
4	and	4	are	8
40	and	40	are	80
5	and	5	are	10
50	and	50	are	100

M.

10	and	5	are	15
	and	5	are	20
	and	5	are	25
	and	5	are	30
	and	5	are	35
	and	5	are	40
	and	5	are	45
	and	5	are	50
	and	5	are	55
	and	5	are	60

N.

5	and	5	are	10
	and	10	are	20
	and	10	are	30
	and	10	are	40
	and	10	are	50
	and	10	are	60
	and	10	are	70
	and	10	are	80
	and	10	are	90
	and	10	are	100

O.

5	and	10	are	15
	and	10	are	25
	and	10	are	35
	and	10	are	45
	and	10	are	55
	and	10	are	65
	and	10	are	75
	and	10	are	85
	and	10	are	95
	and	10	are	105

LESSON TENTH.

REPEAT DIVISION A. OF THE TABLE.

One and cipher are how many? How do you know?
One and one are how many? How do you know?
One and two are how many? How do you know?
Two and three are how many? How do you know?
Three and two are how many? How do you know?
Two and four are how many? How do you know?
Two and six are how many? How do you know?
Seven and two are how many? How do you know?
Two and nine are how many? How do you know?
Ten and two are how many? How do you know?

LESSON ELEVENTH.

REPEAT DIVISION B.

Three and one are how many? How do you know?*
Three and four are how many? Three and six are how many?
Eight and three are how many? Three and ten are how many?
Eleven and three are how many? Seven and three are how many?
Four and three are how many? Twelve and three are how many?

LESSON TWELFTH.

REPEAT DIVISION C.

Four and six are how many? Four and four are how many?

* *The repetition* of this question must be left to the discretion of the teacher.

Four and seven are how many? Nine and four are how many?

Four and eight are how many? Four and ten are how many?

Four and three are how many? Twelve and four are how many?

Four and eleven are how many? Four and five are how many?

LESSON THIRTEENTH.

REPEAT DIVISION D.

Five and two are how many? Three and five are how many?

Five and four are how many? Seven and five are how many?

Five and eight are how many? Nine and five are how many?

Five and ten are how many? Twelve and five are how many?

Five and eleven are how many? Five and five are how many?

LESSON FOURTEENTH.

REPEAT DIVISION E.

Six and three are how many? Six and five are how many?

Seven and six are how many? Six and nine are how many?

Eight and six are how many? Ten and six are how many?

Six and eleven are how many?

LESSON FIFTEENTH.

REPEAT DIVISION F.

Seven and five are how many? Seven and two are how many?

Seven and four are how many? Three and seven are how many?

Seven and six are how many? Seven and eight are how many?

Ten and seven are how many? Seven and twelve are how many?

Nine and seven are how many? Seven and eleven are how many?

LESSON SIXTEENTH.

REPEAT DIVISION G.

Eight and three are how many? Five and eight are how many?

Two and eight are how many? Eight and four are how many?

Eight and six are how many? Three and eight are how many?

Eight and eleven are how many? Ten and eight are how many?

Twelve and eight are how many? Eight and eight are how many?

LESSON SEVENTEENTH.

REPEAT DIVISION H.

Nine and four are how many? Six and nine are how *many?*

Nine and five are how many? Seven and nine are how many?

Nine and nine are how many? Eleven and nine are how many?

Nine and two are how many? Nine and eight are how many?

Nine and twelve are how many? Nine and three are how many?

LESSON EIGHTEENTH.

REPEAT DIVISION I.

Ten and eleven are how many? Ten and eight are how many?

Ten and six are how many? Ten and nine are how many?

Ten and two are how many? Ten and seven are how many?

Ten and four are how many? Three and ten are how many?

Ten and five are how many? Ten and twelve are how many?

LESSON NINETEENTH.

REPEAT DIVISION J.

Eleven and two are how many? Eleven and four are how many?

Eleven and six are how many? Eleven and seven are how many?

Eleven and nine are how many? Eleven and eleven are how many?

Eleven and five are how many? Eleven and twelve are how many?

Eleven and eight are how many? Eleven and three are how many?

LESSON TWENTIETH.

REPEAT DIVISION K.

Twelve and five are how many? Six and twelve are how many?

Twelve and eight are how many? Twelve and ten are how many?

Twelve and seven are how many? Twelve and nine are how many?

Twelve and six are how many? Twelve and eleven are how many?

Twelve and four are how many? Twelve and twelve are how many?

LESSON TWENTY-FIRST.

REPEAT DIVISION L.

Ten and ten are how many? Twenty and twenty are how many?

Thirty and thirty are how many? Forty and forty are how many?

Fifty and fifty are how many?

REPEAT DIVISION M.

Fifteen and five are how many? Twenty-five and five *are how* many?

Forty and five are how many? Thirty and five are how many?

Fifty and five are how many? Forty-five and five are how many?

LESSON TWENTY-SECOND.

REPEAT DIVISION N.

Twenty and ten are how many? Thirty and ten are how many?

Fifty and ten are how many? Seventy and ten are how many?

Sixty and ten are how many? Eighty and ten are how many?

Ninety and ten are how many? Forty and ten are how many?

REPEAT DIVISION O.

Fifteen and ten are how many? Thirty-five and ten are how many?

Fifty-five and ten are how many? Forty-five and ten are how many?

Sixty-five and ten are how many? Eighty-five and ten are how many?

Seventy-five and ten are how many? Ninety-five and ten are how many?

LESSON TWENTY-THIRD.

NOTE.—Let the teacher read the question, and the scholar answer it. The teacher will then require the process, which the scholar will give according to the example.

QUEST. 1.—John spent three cents for candy, four for

raisins, and two for marbles; how many cents did he spend in all?

S. Nine.

T. Give the process.

S. Three and four are seven, and two are nine.

2.—Peter gave three cents to one poor boy, five to another, and four to another; how many did he give away in all?

Give the process.

3.—Joseph's personal property consisted in a jackknife, valued at eight cents, a money-purse, valued at seven cents, and a whip, valued at six cents; what was the whole value of his possessions?

Give the process.

4.—Elisha had nine cents, his father gave him eight more, and his mother gave him 6; how many then had he?

Give the process.

5.—Julia had five rings, her mother gave her three more, and her sister gave her four more; how many then had she?

Give the process.

LESSON TWENTY-FOURTH.

QUEST. 1.—Jennett had seven books, she purchased five more; she received as a present six more; how many then had she?

Give the process.

2.—Emily purchased a story-book for nine cents, a doll for eleven cents, and a thimble for five cents; what did she pay in all?

Give the process.

3.—Enoch had ten doves, and ten more came to his flock at one time, and six at another; how many then had he?

Give the process.

4.—Jeremiah had twelve marbles; he gained of one boy by playing seven more, of another, eight; how many then had he?

Give the process.

5.—Stephen purchased a slate for thirteen cents, a penknife for eleven, and some pens for nine; how many cents did he pay for the whole?

LESSON TWENTY-FIFTH.

QUEST. 1.—A man travels fourteen miles on Monday, eleven on Tuesday, and twelve on Wednesday; how many miles does he travel in all?

Give the process.

2.—A man purchases a barrel of flour for thirteen dollars, fifteen bushels of corn for fifteen dollars, and a barrel of molasses for nine dollars; what does he pay for the whole?

Give the process.

3.—A man purchases a yard of cloth for nine dollars, three pairs of shoes for five dollars, a hat for four dollars, and a vest for six dollars; what does he pay for the whole?

Give the process.

4.—A boy pays seven cents for oranges, five cents for lemons, six cents for paper, and eleven cents for crackers; what does he pay in all?

Give the process.

LESSON TWENTY-SIXTH.

QUEST. 1.—Fourteen men are employed on one section of a railroad, five on another, six on another, and seven on another; how many are employed in all?

Give the process.

2.—Four families reside in one house; one family consists of nine persons, one of eight, one of seven, and one of eleven; how many persons are there in all?

Give the process.

3.—If I purchase one acre of land for thirteen dollars, another for fourteen, another for ten, and another for nine, how many dollars do I give for the whole?

Give the process.

4.—In a certain school eleven scholars study arithmetic, fifteen grammar, five geography, and six study philosophy; how many are there in all these classes?

Give the process.

LESSON TWENTY-SEVENTH.

1. Nine and two are how many?
2. Nine and three are how many?
3. Nine and four are how many?
4. Nine and six are how many?
5. Nine and nine are how many?
6. Nine and ten are how many?

Ten and ten are twenty. Nineteen is one less than twenty, and nine is one less than ten; therefore nine and ten are nineteen.

7. Nineteen and nine are how many?
8. Twenty-nine and nine are how many?
9. Thirty-nine and nine are how many?
10. Forty-nine and nine are how many?
11. Fifty-nine and nine are how many?
12. Sixty-nine and nine are how many?
13. Seventy-nine and nine are how many?
14. Eighty-nine and nine are how many?
15. Ninety-nine and nine are how many?

LESSON TWENTY-EIGHTH.

1. Seven and nine are how many?
2. Seventeen and nine are how many?
3. Twenty-seven and nine are how many?
4. Thirty-seven and nine are how many?
5. Nineteen and seven are how many?
6. Twenty-nine and seven are how many?
7. Twenty-five and nine are how many?
8. Twenty-nine and five are how many?
9. Thirty-five and nine are how many?
10. Thirty-nine and five are how many?
11. Thirty-nine and nine are how many?
12. Thirty-five and nine are how many?
13. Eighty-nine and nine are how many?
14. Forty-four and seven are how many?

LESSON TWENTY-NINTH.

1. Forty-five and six are how many?
2. Forty-six and seven are how many?
3. Forty-eight and eight are how many?
4. Forty-nine and nine are how many?
5. Fifty-five and five are how many?
6. Fifty-six and six are how many?
7. Fifty-eight and eight are how many?
8. Fifty-nine and nine are how many?
9. Fifty-four and nine are how many?
10. Sixty-three and five are how many?
11. Sixty-six and seven are how many?
12. Sixty-seven and eight are how many?
13. Sixty-eight and six are how many?
14. Sixty-nine and four are how many?

SECTION II.

LESSON FIRST.

Teacher. One and one are how many?
Scholar. Two.
T. Then one from two how many remain?
S. One.
T. How do you know?
S. One and one are two, therefore one from two leaves
T. Two and one are how many?
S. Three.
T. Then one from three, how many remain?
S. Two.
T. How do you know?
S. Two and one are three, therefore one from three le two.
T. Two from three, how many remain?
S. ———
T. How do you know?
S. ———
T. One from four, how many remain?
S. ———
T. Two from four, how many remain?
S. ———
T. How do you know?
S. ———
T. Three from five, how many remain?
S. ———
T. How do you know?
S. ———
T. Are the questions in this lesson similar to those in preceding lessons?
S. They are not.
T. In what respect do they differ?
S. They are the reverse.

T. Illustrate what you mean by "the reverse."

S. If, when we have added two to three, which are five, we take two from five, we reverse the process.

T. What is the latter process called?

S. Subtraction.

T. What, then, is Subtraction?

S. It is finding the difference between two numbers?

LESSON SECOND.

QUEST. 1.—Mary had two cents, and she gave away one of them; how many had she left?

One from two, how many remain?

How do you know?

2.—James had three cents, and he gave away one of them; how many had he left?

One from three, and how many remain?

How do you know?

3.—Jane had four cents, and she gave her sister two of them; how many had she left?

Two from four, and how many remain?

How do you know?

4.—Julia had five apples, and she gave her sister two of them; how many had she left?

Two from five, and how many remain?

How do you know?

5.—Joseph had six apples, and he gave away three of them; how many had he left?

Three from six, and how many remain?

How do you know?

LESSON THIRD.

QUEST. 1.—John had seven peaches; he gave away three of them; how many had he left?

Three from seven, and how many remain?
How do you know?

2.—Charles had eight cents, and he gave away three of them; how many had he left?
Three from eight, and how many remain?
How do you know?

3.—Thomas had nine nuts, and he gave away four of them; how many had he left?
Four from nine, and how many remain?
How do you know?

4.—William had ten apples, and gave away five of them; how many had he left?
Five from ten, and how many remain?
How do you know?

5.—George had eleven dollars, and he gave away five of them; how many had he left?
Five from eleven, and how many remain?
How do you know?

6.—Isaac had twelve doves, and he gave away six of them; how many had he left?
Six from twelve, and how many remain?
How do you know?

SUBTRACTION TABLE.

A.					B.				
1	from	1	leaves	0	2	from	2	leaves	0
1	from	2	leaves	1	2	from	3	leaves	1
1	from	3	leaves	2	2	from	4	leaves	2
1	from	4	leaves	3	2	from	5	leaves	3
1	from	5	leaves	4	2	from	6	leaves	4
1	from	6	leaves	5	2	from	7	leaves	5
1	from	7	leaves	6	2	from	8	leaves	6
1	from	8	leaves	7	2	from	9	leaves	7
1	from	9	leaves	8	2	from	10	leaves	8
1	from	10	leaves	9	2	from	11	leaves	9
1	from	11	leaves	10	2	from	12	leaves	10
1	from	12	leaves	11	2	from	13	leaves	11
1	from	13	leaves	12	2	from	14	leaves	12

C.

3 from 3 leaves 0
3 from 4 leaves 1
3 from 5 leaves 2
3 from 6 leaves 3
3 from 7 leaves 4
3 from 8 leaves 5
3 from 9 leaves 6
3 from 10 leaves 7
3 from 11 leaves 8
3 from 12 leaves 9
3 from 13 leaves 10
3 from 14 leaves 11
3 from 15 leaves 12

D.

4 from 4 leaves 0
4 from 5 leaves 1
4 from 6 leaves 2
4 from 7 leaves 3
4 from 8 leaves 4
4 from 9 leaves 5
4 from 10 leaves 6
4 from 11 leaves 7
4 from 12 leaves 8
4 from 13 leaves 9
4 from 14 leaves 10
4 from 15 leaves 11
4 from 16 leaves 12

E.

5 from 5 leaves 0
5 from 6 leaves 1
5 from 7 leaves 2
5 from 8 leaves 3
5 from 9 leaves 4
5 from 10 leaves 5
5 from 11 leaves 6
5 from 12 leaves 7
5 from 13 leaves 8
5 from 14 leaves 9
5 from 15 leaves 10
5 from 16 leaves 11
5 from 17 leaves 12

F.

6 from 6 leaves 0
6 from 7 leaves 1
6 from 8 leaves 2
6 from 9 leaves 3
6 from 10 leaves 4
6 from 11 leaves 5
6 from 12 leaves 6
6 from 13 leaves 7
6 from 14 leaves 8
6 from 15 leaves 9
6 from 16 leaves 10
6 from 17 leaves 11
6 from 18 leaves 12

G.

7 from 7 leaves 0
7 from 8 leaves 1
7 from 9 leaves 2
7 from 10 leaves 3
7 from 11 leaves 4
7 from 12 leaves 5
7 from 13 leaves 6
7 from 14 leaves 7
7 from 15 leaves 8
7 from 16 leaves 9
7 from 17 leaves 10
7 from 18 leaves 11
7 from 19 leaves 12
7 from 20 leaves 13

H.

8	from	8	leaves		0
8	from	9	leaves		1
8	from	10	leaves		2
8	from	11	leaves		3
8	from	12	leaves		4
8	from	13	leaves		5
8	from	14	leaves		6
8	from	15	leaves		7
8	from	16	leaves		8
8	from	17	leaves		9
8	from	18	leaves		10
8	from	19	leaves		11
8	from	20	leaves		12

I.

9	from	9	leaves		0
9	from	10	leaves		1
9	from	11	leaves		2
9	from	12	leaves		3
9	from	13	leaves		4
9	from	14	leaves		5
9	from	15	leaves		6
9	from	16	leaves		7
9	from	17	leaves		8
9	from	18	leaves		9
9	from	19	leaves		10
9	from	20	leaves		11
9	from	21	leaves		12

J.

10	from	10	leaves		0
10	from	11	leaves		1
10	from	12	leaves		2
10	from	13	leaves		3
10	from	14	leaves		4
10	from	15	leaves		5
10	from	16	leaves		6
10	from	17	leaves		7
10	from	18	leaves		8
10	from	19	leaves		9
10	from	20	leaves		10
10	from	21	leaves		11
10	from	22	leaves		12

K.

11	from	11	leaves		0
11	from	12	leaves		1
11	from	13	leaves		2
11	from	14	leaves		3
11	from	15	leaves		4
11	from	16	leaves		5
11	from	17	leaves		6
11	from	18	leaves		7
11	from	19	leaves		8
11	from	20	leaves		9
11	from	21	leaves		10
11	from	22	leaves		11
11	from	23	leaves		12

L.

12	from	12	leaves		0
12	from	13	leaves		1
12	from	14	leaves		2
12	from	15	leaves		3
12	from	16	leaves		4
12	from	17	leaves		5
12	*from*	*18*	leaves		6
12	from	19	leaves		7
12	from	20	leaves		8
12	from	21	leaves		9
12	from	22	leaves		10
12	from	23	leaves		11
12	from	24	leaves		12
12	from	25	leaves		13

LESSON FOURTH.

Obs.—The following seven lessons are intended to embrace each a lesson to be repeated from the Table, before the questions are proposed.

One from two leaves how many? One from three leaves how many?

One from five leaves how many? Two from three leaves how many?

Two from five leaves how many? Two from seven leaves how many?

Two from nine leaves how many? Two from twelve leaves how many?

Two from eleven leaves how many? Three from three leaves how many?

Three from five leaves how many? Three from seven leaves how many?

Three from six leaves how many? Three from nine leaves how many?

Three from twelve leaves how many?

LESSON FIFTH.

Four from six leaves how many? Four from nine leaves how many?

Four from seven leaves how many? Four from ten leaves how many?

Four from eight leaves how many? Four from eleven leaves how many?

Four from five leaves how many? Five from twelve leaves how many?

Five from five leaves how many? Five from seven leaves how many?

Five from ten leaves how many? Five from nine leaves how many?

Five from eleven leaves how many? Five from eight leaves how many?

Five from six leaves how many?

LESSON SIXTH.

Six from seven leaves how many? Six from eight leaves how many?

Six from ten leaves how many? Six from twelve leaves how many?

Six from fourteen leaves how many? Six from seventeen leaves how many?

Six from thirteen leaves how many? Six from eighteen leaves how many?

Six from fifteen leaves how many? Six from nine leaves how many?

Seven from ten leaves how many? Seven from thirteen leaves how many?

Seven from twelve leaves how many? Seven from seventeen leaves how many?

Seven from eleven leaves how many? Seven from fifteen leaves how many?

Seven from eighteen leaves how many? Seven from eight leaves how many?

LESSON SEVENTH.

Eight from eight leaves how many? Eight from ten leaves how many?

Eight from thirteen leaves how many? Eight from eleven leaves how many?

Eight from fifteen leaves how many? Eight from twelve leaves how many?

Eight from nine leaves how many? Eight from sixteen leaves how many?

Eight from eighteen leaves how many? Eight from twenty leaves how many?

Eight from seventeen leaves how many? Eight from nineteen leaves how many?

Eight from fourteen leaves how many? Eight from *twenty leaves* how many?

LESSON EIGHTH.

Nine from twenty-one leaves how many? Nine from eighteen leaves how many?

Nine from sixteen leaves how many? Nine from twelve leaves how many?

Nine from ten leaves how many? Nine from thirteen leaves how many?

Nine from fifteen leaves how many? Nine from seventeen leaves how many?

Nine from eleven leaves how many? Nine from nineteen leaves how many?

Nine from twenty leaves how many? Nine from fourteen leaves how many?

LESSON NINTH.

Ten from twenty leaves how many? Ten from twenty-two leaves how many?

Ten from nineteen leaves how many? Ten from seventeen leaves how many?

Ten from fifteen leaves how many? Ten from thirteen leaves how many?

Ten from twenty-one leaves how many? Ten from twelve leaves how many?

Ten from eleven leaves how many? Ten from eighteen leaves how many?

Ten from sixteen leaves how many? Ten from thirteen leaves how many?

Ten from fourteen leaves how many?

LESSON TENTH.

Eleven from fifteen leaves how many? Eleven from seventeen leaves how many?

Eleven from thirteen leaves how many? Eleven from eighteen leaves how many?

Eleven from twenty leaves how many? Eleven from sixteen leaves how many?

Eleven from twenty-one leaves how many? Eleven from twenty-three leaves how many?

Eleven from fourteen leaves how many? Eleven from nineteen leaves how many?

Eleven from twenty-two leaves how many? Eleven from twelve leaves how many?

LESSON ELEVENTH.

Twelve from twenty-three leaves how many?
Twelve from nineteen leaves how many?
Twelve from fourteen leaves how many?
Twelve from fifteen leaves how many?
Twelve from thirteen leaves how many?
Twelve from twenty-one leaves how many?
Twelve from twenty-two leaves how many?
Twelve from sixteen leaves how many?
Twelve from twenty-four leaves how many?
Twelve from twenty leaves how many?
Twelve from seventeen leaves how many?
Twelve from eighteen leaves how many?
Twelve from twelve leaves how many?

LESSON TWELFTH.

QUEST. 1.—John earned three cents in one day, cents in another, five in another; in the mean time he six. How many had he left?

T. Give the process.

S. Three and four are seven, and five are twelv *from twelve and* six remain.

2.—Philip had nine cents, Josiah had seven, Joel had five; how many had they all; and how many more had Philip and Josiah than Joel?

Give the process.

3.—Frank had eleven pears, and he gave four of them to his companion; how many had he left; and how many more had he than his companion?

4.—Jonathan had ten apples, Joseph eleven, and James fourteen: Jonathan gave four to Timothy, Joseph gave him five, and James gave him six; how many had Timothy, and how many had each left?

Give the process.

LESSON THIRTEENTH.

QUEST. 1.—A man had twenty dollars, and he gave to one poor boy four dollars, to another five, to another six; how many dollars did he gave away, and how many had he left?

Give the process.

2.—A man gave to three charitable purposes the following sums: to one sixteen dollars, to another fifteen, to another twelve, and reserved ten dollars for his own use; how many did he give away, and how many had he at first?

3.—A boy gave twelve cents for a slate, three cents for pencils, six cents for oranges, seven cents for gingerbread, and had nine cents left; how many cents did he pay away, and how many had he at first?

4.—A boy gave ten cents for buffalo skin to make a pair of mittens, and a bushel of ashes, worth eight cents, for the making: he sold them for twenty cents; what did his mittens cost him, and how much did he gain by his bargain?

LESSON FOURTEENTH.

QUEST. 1.—A vessel sails thirteen miles the first hour, and is carried back by an under-current three miles; the second hour it sails fourteen miles, and is carried back four; how many miles does it gain in the two hours?

Give the process.

2.—A boy is to receive twenty cents a day, for one week, for every day he works, and is to pay his employer ten cents a day for every day he plays: he works four days and plays three; how many cents has he at the end of the week?

3.—A man purchases a horse for twenty-five dollars, and a wagon for eighteen; how much does he give for the horse and wagon; and how much more for the horse than the wagon?

4.—A man purchases a tub of butter for seventeen dollars, a hundred pounds of sugar for nine dollars, a hundred pounds of coffee for eleven dollars; how much does he pay for the whole; and how much more for the sugar and coffee than for the butter?

5.—A pound of hops costs ten cents, and a pound of tea fifty, and a pound of rice eight; what cost the whole; and how much more does the tea cost than the hops and the rice?

LESSON FIFTEENTH.

1.—A. builds twelve rods of wall, B. builds ten, C. builds seven, and D. builds nine; how many rods do they all build; and how many more do A. and B. build than C. and D.?

2.—A. travels eighteen miles, B. travels sixteen, C. travels fourteen, and D. travels nine; how many miles do they all travel; and how many miles more do A. and B. travel than *C. and D.?*

Eighteen less fifteen are how many?
Twenty-five less nine are how many?
Thirty-five less six are how many?
Forty-five less five are how many?
Fifty-five less ten are how many?

LESSON SIXTEENTH.

1. Ten less five are how many?
2. Fifteen less five are how many?
3. Fifteen less ten are how many?
4. Twenty less five are how many?
5. Twenty less ten are how many?
6. Twenty less fifteen are how many?
7. Twenty-five less fifteen are how many?
8. Twenty-five less ten are how many?
9. Thirty less fifteen are how many?
10. Thirty-five less ten are how many?
11. Forty less fifteen are how many?
12. Forty-five less fifteen are how many?

LESSON SEVENTEENTH.

1. Forty-five less twenty are how many?
2. Fifty less fifteen are how many?
3. Fifty less twenty-five are how many?
4. Fifty-five less fifteen are how many?
5. Fifty-five less twenty-five are how many?
6. Sixty-five less twenty-five are how many?
7. Seventy-five less twenty are how many?
8. Eighty-five less fifteen are how many?
9. Eighty-five less twenty are how many?
10. Ninety-five less fifteen are how many?
11. Ninety-five less twenty are how many?
12. One hundred less twenty-five are how many?
13. One hundred less twenty are how many?

SECTION III.

LESSON FIRST.

Teacher. One and one are how many?
Scholar. Two.
T. Then twice one are how many?
S. Two.
T. How do you know?
S. Because one and one are two; therefore, twi
are two.
T. Twice two are how many?
S. Four.
T. How do you know?
S. Because two and two are four; therefore, twi
are four.
T. Three times one are how many?
S. Three.
T. How do you know?
S. Because one and one and one are three, which
three times repeated; therefore, three times one are t
T. Twice three are how many?
S. Six.
T. How do you know?
S. Because three and three are six; therefore, twic
are six.
T. Three times two are how many?
S. Six.
T. How do you know?
S. Because two and two and two are six; therefor
times two are six.
T. Are three times two the same as twice three?
S. They are.
T. How do you know?
S. Because they are the same numbers repeated i
versed order.

LESSON SECOND.

T. Twice four are how many?

S. Eight.

T. Twice five are how many?

S. Ten.

T. How do you know?

S. Because five and five are ten; therefore twice five are ten?

T. Three times four are how many?

S. Twelve.

T. How do you know?

S. Because four and four and four are twelve, which is four three times repeated; therefore, three times four are twelve.

T. Which is the shorter process, to say three times four are twelve, or to add four to itself two times?

S. To say three times four are twelve.

T. What is this latter process called?

S. Multiplication.

T. What, then, is multiplication?

S. It is a shorter mode of performing addition.

T. Twice six are how many?

S. Twelve.

T. How do you know?

S. Because six and six are twelve, which is six twice repeated, therefore twice six are twelve.

T. Are twice six, the same as three times four?

S. They are.

T. How do you know?

S. Because three times four are twelve, and twice six are twelve; being equal to the same, they are equal to one another.

Let the teacher illustrate this last clause.

LESSON THIRD.

QUEST. 1.—Edward bought two sticks of candy for two cents a stick; what did he pay for the two?

If for one stick he paid two cents, for two sticks he would pay twice as much, and twice two are four; therefore, for two sticks he would pay four cents.

2.—Myron purchased three sheets of paper for three cents a sheet; what did he pay for the three sheets?

Give the process.

3.—Joseph purchased four peaches for four cents apiece; what did he pay for the four?

Give the process.

4.—Olive Ann purchased four combs, at five cents apiece; what did she pay for the four?

Give the process.

5.—Benjamin purchased five combs, at four cents apiece; what did he pay for the five?

Give the process.

6.—Are five times four the same as four times five?

How do you know?

LESSON FOURTH.

QUEST. 1.—Julia purchased six skeins of silk, at four cents a skein; what did she pay for the four skeins?

Give the process.

2.—Charles purchased a slate for eight cents, and a dozen pencils for six cents; what did he pay for the whole?

T. Can this question be performed by multiplication?

S. It cannot.

T. Why not?

S. Because we do not wish to repeat either the 8 or the 6, but only to add the one to the other.

T. Then all questions in addition cannot be performed by multiplication?

S. They cannot.

T. What questions can be?

S. Such questions as require a number to be repeated a certain number of times.

T. Can all questions in multiplication be performed by addition?

S. They can.

T. In what, then, does multiplication consist?

S. In the repetition of a given number a required number of times.

MULTIPLICATION TABLE.

A.					B.				
2	times	1	are	2	3	times	1	are	3
2	times	2	are	4	3	times	2	are	6
2	times	3	are	6	3	times	3	are	9
2	times	4	are	8	3	times	4	are	12
2	times	5	are	10	3	times	5	are	15
2	times	6	are	12	3	times	6	are	18
2	times	7	are	14	3	times	7	are	21
2	times	8	are	16	3	times	8	are	24
2	times	9	are	18	3	times	9	are	27
2	times	10	are	20	3	times	10	are	30
2	times	11	are	22	3	times	11	are	33
2	times	12	are	24	3	times	12	are	36

C.					D.				
4	times	1	are	4	5	times	1	are	5
4	times	2	are	8	5	times	2	are	10
4	times	3	are	12	5	times	3	are	15
4	times	4	are	16	5	times	4	are	20
4	times	5	are	20	5	times	5	are	25
4	times	6	are	24	5	times	6	are	30
4	times	7	are	28	5	times	7	are	35
4	times	8	are	32	5	times	8	are	40
4	times	9	are	36	5	times	9	are	45
4	times	10	are	40	5	times	10	are	50
4	times	11	are	44	5	times	11	are	55
4	times	12	are	48	5	times	12	are	60

E.					F.				
6	times	1	are	6	7	times	1	are	7
6	times	2	are	12	7	times	2	are	14
6	times	3	are	18	7	times	3	are	21
6	times	4	are	24	7	times	4	are	28
6	times	5	are	30	7	times	5	are	35
6	times	6	are	36	7	times	6	are	42
6	times	7	are	42	7	times	7	are	49
6	times	8	are	48	7	times	8	are	56
6	times	9	are	54	7	times	9	are	63
6	times	10	are	60	7	times	10	are	70
6	times	11	are	66	7	times	11	are	77
6	times	12	are	72	7	times	12	are	84

G.					H.				
8	times	1	are	8	9	times	1	are	9
8	times	2	are	16	9	times	2	are	18
8	times	3	are	24	9	times	3	are	27
8	times	4	are	32	9	times	4	are	36
8	times	5	are	40	9	times	5	are	45
8	times	6	are	48	9	times	6	are	54
8	times	7	are	56	9	times	7	are	63
8	times	8	are	64	9	times	8	are	72
8	times	9	are	72	9	times	9	are	81
8	times	10	are	80	9	times	10	are	90
8	times	11	are	88	9	times	11	are	99
8	times	12	are	96	9	times	12	are	108

I.					J.				
10	times	1	are	10	11	times	1	are	11
10	times	2	are	20	11	times	2	are	22
10	times	3	are	30	11	times	3	are	33
10	times	4	are	40	11	times	4	are	44
10	times	5	are	50	11	times	5	are	55
10	times	6	are	60	11	times	6	are	66
10	times	7	are	70	11	times	7	are	77
10	times	8	are	80	11	times	8	are	88
10	times	9	are	90	11	times	9	are	99
10	times	10	are	100	11	times	10	are	110
10	*times*	11	are	110	11	times	11	are	121
10	*times*	12	are	120	11	times	12	are	132

K.

12	times	1	are	12
12	times	2	are	24
12	times	3	are	36
12	times	4	are	48
12	times	5	are	60
12	times	6	are	72
12	times	7	are	84
12	times	8	are	96
12	times	9	are	108
12	times	10	are	120
12	times	11	are	132
12	times	12	are	144

LESSON FIFTH.

REPEAT DIVISION A. OF THE TABLE.

1. Twice two are how many?
2. Twice four are how many?
3. Twice six are how many?
4. Twice seven are how many?
5. Twice eight are how many?
6. Seven times two are how many?
7. Are seven times two, the same as two times seven?
8. Twice nine are how many?
9. Twice ten are how many?
10. Eleven times two are how many?
11. Twice twelve are how many?
12. Three times two are how many?

REPEAT DIVISION B.

13. Three times one are how many?
14. Three times four are how many?
15. Three times six are how many?
16. Five times three are how many?
17. Three times seven are how many?
18. Seven times three are how many?

19. Three times eight are how many?
20. Three times eleven are how many?
21. Twelve times three are how many?
22. Ten times three are how many?

LESSON SIXTH.

REPEAT DIVISION C.

1. Four times three are how many?
2. Four times five are how many?
3. Six times four are how many?
4. Four times seven are how many?
5. Eight times four are how many?
6. Four times nine are how many?
7. Eleven times four are how many?
8. Four times ten are how many?
9. Four times twelve are how many?

REPEAT DIVISION D.

10. Five times five are how many?
11. Five times three are how many?
12. Six times five are how many?
13. Five times seven are how many?
14. Eight times five are how many?
15. Five times four are how many?
16. Five times nine are how many?
17. Eleven times five are how many?
18. Five times twelve are how many?
19. Five times ten are how many?

LESSON SEVENTH.

REPEAT DIVISION E.

1. Six times two are how many?
2. *Three* times six are how many?

3. Six times four are how many?
4. Six times six are how many?
5. Five times six are how many?
6. Six times seven are how many?
7. Six times nine are how many?
8. Eleven times six are how many?
9. Six times ten are how many?
10. Twelve times six are how many?

REPEAT DIVISION F.

11. Seven times seven are how many?
12. Seven times three are how many?
13. Eight times seven are how many?
14. Seven times ten are how many?
15. Eleven times seven are how many?
16. Seven times five are how many?
17. Seven times nine are how many?
18. Seven times twelve are how many?
19. Seven times two are how many?
20. Seven times six are how many?

LESSON EIGHTH.

REPEAT DIVISION G.

1. Eight times twelve are how many?
2. Eight times eight are how many?
3. Eight times six are how many?
4. Nine times eight are how many?
5. Eight times four are how many?
6. Eight times ten are how many?
7. Seven times eight are how many?
8. Eight times five are how many?
9. Eleven times eight are how many?
10. Eight times seven are how many?

REPEAT DIVISION H.

11. Nine times seven are how many?
12. Nine times five are how many?
13. Eight times nine are how many?
14. Nine times eight are how many?
15. Nine times nine are how many?
16. Eleven times nine are how many?
17. Nine times four are how many?
18. Nine times six are how many?
19. Nine times twelve are how many?
20. Eleven times nine are how many?

LESSON NINTH.

REPEAT DIVISION I.

1. Ten times two are how many?
2. Ten times four are how many?
3. Ten times ten are how many?
4. Ten times seven are how many?
5. Ten times nine are how many?
6. Ten times six are how many?
7. Ten times eight are how many?
8. Ten times ten are how many?
9. Ten times twelve are how many?
10. Eleven times ten are how many?

REPEAT DIVISION J.

11. Eleven times two are how many?
12. Three times eleven are how many?
13. Eleven times five are how many?
14. Eleven times seven are how many?
15. Eight times eleven are how many?
16. *Eleven* times ten are how many?

17. Eleven times twelve are how many?
18. Eleven times nine are how many?
19. Eleven times eleven are how many?
20. Ten times eleven are how many?

LESSON TENTH.

REPEAT DIVISION K.

1. Twelve times five are how many?
2. Twelve times three are how many?
3. Twelve times six are how many?
4. Seven times twelve are how many?
5. Twelve times nine are how many?
6. Twelve times four are how many?
7. Twelve times ten are how many?
8. Twelve times two are how many?
9. Twelve times eleven are how many?
10. Twelve times twelve are how many?

11. If a boy pay two cents for one picture-book, how many cents must he pay for four? How many for six? How many for seven? How many for five?

If one picture-book cost two cents, four picture-books will cost four times as many cents, and four times two are eight; therefore, four picture-books will cost eight cents.

12. If a picture-frame cost twelve dollars, how many dollars will four cost? How many six? How many seven? How many eight? How many nine?

LESSON ELEVENTH.

QUEST. 1.—Four men bought a piece of land, each paying twelve dollars; what did they all pay?

2.—If a horse trot eleven miles in one hour, how many miles will he trot in eight hours? In ten hours? In eleven hours? In twelve hours?

3.—If a bushel of wheat cost two dollars, how many dollars will eight bushels cost? How many nine bushels? How many eleven bushels? How many twelve bushels?

4.—If a man receive four shillings for one day's work, how many shillings will he receive for six days' work? How many for seven? How many for nine? How many for eleven? How many for twelve?

5.—If a pound of sugar cost eight cents, how many cents will five pounds cost? How many seven? How many nine? How many ten? How many eleven? How many twelve?

6.—John bought a writing-book for six cents, how many cents would four cost? How many six? How many eight? How many ten? How many twelve?

LESSON TWELFTH.

QUEST. 1.—If a bushel of potatoes cost twenty-five cents, how many cents will two bushels cost? How many four?

2.—If a bushel of apples cost fifty cents, how many cents will two bushels cost? How many four?

3.—If a man travel twenty miles in one day, how many miles will he travel in five days? How many in ten days?

4.—A steamboat makes progress ten miles an hour, how many miles will it make in four hours? How many in five? How many in six? How many in seven? How many in eight? How many in nine? How many in ten? How many in twelve? How many in eleven?

SECTION IV.

LESSON FIRST.

Teacher. Twice one are how many?
Scholar. Two.
T. Then one is contained in two how many times?
S. Twice.
T. How do you know?
S. Because twice one are two.
T. Two is contained in two how many times?
S. Once.
T. How do you know?
S. Because any number must contain itself once.
T. How many more times is one contained in two, than two in two?
S. One.
T. Why?
S. Because one is half as large as two, therefore it would be contained in two, twice as many times as two would be contained in two.
T. Two is contained in four how many times?
S. Twice.
T. How do you know?
S. Because twice two are four.
T. Three is contained in six how many times?
S. Twice.
T. How do you know?
S. Because twice three are six.
T. Four is contained in eight how many times?
S. Twice.
T. How do you know?
S. Because twice four are eight.
T. Two is contained in eight how many times?
S. Four times.
T. How do you know?
S. Because four times two are eight.

T. How many more times is two contained in eight, than four in eight?

S. Two times.

T. Why?

S. Because two is half as large as four, therefore it would be contained in any given number twice as many times.

T. What is the number of times one number is contained in another called?

S. Quotient.

LESSON SECOND.

QUEST. 1.—Mary gave four cents for two apples; how many cents did she give for each apple?

S. If two apples cost four cents, one apple will cost half as much; one half of four is two; one apple, therefore, will cost two cents.

T. How do you know that one apple will cost half as much as two?

S. I know, because one is half of two; therefore the cost of one apple will be half the cost of two apples.

2.—John gave six cents for three peaches; how many cents did he pay for one peach?

Give the process.

T. What is the process called?

S. Division.

T. Is it similar to any you have had before?

S. It is the same as Subtraction, only a shorter process.

T. Can all questions in Subtraction be performed by Division?

S. They cannot.

T. What questions can?

S. Such as are to be divided into equal parts.

3.—Jane had eight apples; she wished to divide them *equally between* herself and her sister Lucy. How may she *do it by Subtraction*?

S. She may take one for herself, and give one to her sister, and take another for herself, and give another to her sister, and so continue until all are distributed, and this would be done by Subtraction.

T. How may she do it by Division?

S. Divide 8 by 2, and the quotient would be the part of each.

T. Which would be the shorter mode?

S. The latter.

T. What, then, is Division?

S. It is a shorter mode of performing Subtraction.

LESSON THIRD.

QUEST. 1.—James gave six cents for two peaches; how many cents did he give for each peach?

Give the process.

Why is two contained in six one time more than three?

2. John pays nine cents for three spools of cotton; how many cents does he pay for one spool?

Give the process.

3.—Henry had five oranges given him; he kept one himself, and divided the others between his two sisters; how many did each of his sisters receive?

Give the solution.

What two processes do you employ in solving this question?

4.—James's father gave him eight butternuts, to divide equally between himself and his three brothers; how many did each receive?

5.—Harriet paid twenty-four cents for six pictures; how many cents did she pay for each one?

6.—Mary divided thirty-six cents equally between six poor children; how many cents did each receive?

7.—How many yards of cloth can be bought for thirty-two dollars, at the rate of eight dollars per yard?

DIVISION TABLE.

A.					B.				
2	in	2	1	time	3	in	3	1	time
2	in	4	2	times	3	in	6	2	times
2	in	6	3	times	3	in	9	3	times
2	in	8	4	times	3	in	12	4	times
2	in	10	5	times	3	in	15	5	times
2	in	12	6	times	3	in	18	6	times
2	in	14	7	times	3	in	21	7	times
2	in	16	8	times	3	in	24	8	times
2	in	18	9	times	3	in	27	9	times
2	in	20	10	times	3	in	30	10	times
2	in	22	11	times	3	in	33	11	times
2	in	24	12	times	3	in	36	12	times

C.					D.				
4	in	4	1	time	5	in	5	1	time
4	in	8	2	times	5	in	10	2	times
4	in	12	3	times	5	in	15	3	times
4	in	16	4	times	5	in	20	4	times
4	in	20	5	times	5	in	25	5	times
4	in	24	6	times	5	in	30	6	times
4	in	28	7	times	5	in	35	7	times
4	in	32	8	times	5	in	40	8	times
4	in	36	9	times	5	in	45	9	times
4	in	40	10	times	5	in	50	10	times
4	in	44	11	times	5	in	55	11	times
4	in	48	12	times	5	in	60	12	times

E.					F.				
6	in	6	1	time	7	in	7	1	time
6	in	12	2	times	7	in	14	2	times
6	in	18	3	times	7	in	21	3	times
6	in	24	4	times	7	in	28	4	times
6	in	30	5	times	7	in	35	5	times

6	in	36	6	times	7	in	42	6	times
6	in	42	7	times	7	in	49	7	times
6	in	48	8	times	7	in	56	8	times
6	in	54	9	times	7	in	63	9	times
6	in	60	10	times	7	in	70	10	times
6	in	66	11	times	7	in	77	11	times
6	in	72	12	times	7	in	84	12	times

G.

8	in	8	1	time	9	in	9	1	time
8	in	16	2	times	9	in	18	2	times
8	in	24	3	times	9	in	27	3	times
8	in	32	4	times	9	in	36	4	times
8	in	40	5	times	9	in	45	5	times
8	in	48	6	times	9	in	54	6	times
8	in	56	7	times	9	in	63	7	times
8	in	64	8	times	9	in	72	8	times
8	in	72	9	times	9	in	81	9	times
8	in	80	10	times	9	in	90	10	times
8	in	88	11	times	9	in	99	11	times
8	in	96	12	times	9	in	108	12	times

H.

I.

10	in	10	1	time	11	in	11	1	time
10	in	20	2	times	11	in	22	2	times
10	in	30	3	times	11	in	33	3	times
10	in	40	4	times	11	in	44	4	times
10	in	50	5	times	11	in	55	5	times
10	in	60	6	times	11	in	66	6	times
10	in	70	7	times	11	in	77	7	times
10	in	80	8	times	11	in	88	8	times
10	in	90	9	times	11	in	99	9	times
10	in	100	10	times	11	in	110	10	times
10	in	110	11	times	11	in	121	11	times
10	in	120	12	times	11	in	132	11	times

J.

K.

12	in	12	1	time
12	in	24	2	times
12	in	36	3	times
12	in	48	4	times
12	in	60	5	times
12	in	72	6	times
12	in	84	7	times
12	in	96	8	times
12	in	108	9	times
12	in	120	10	times
12	in	132	11	times
12	in	144	12	times

LESSON FOURTH.

Two is contained in eighteen how many times?
Two is contained in fourteen how many times?
Two is contained in twenty how many times?
Five is contained in twenty-five how many times?
Five is contained in twenty how many times?
Three is contained in twenty-four how many times?
Three is contained in twelve how many times?
Three is contained in twenty-seven how many times?
Three is contained in thirty-three how many times?
Three is contained in thirty-six how many times?
Four is contained in sixteen how many times?
Four is contained in twenty-four how many times?
Four is contained in twenty how many times?
Four is contained in thirty-six how many times?
Four is contained in forty-four how many times?
Four is contained in forty-eight how many times?

LESSON FIFTH.

Five is contained in twenty-five how many times?
Five is contained in thirty how many times?
Five is contained in forty how many times?
Five is contained in fifty how many times?
Five is contained in sixty how many times?
Five is contained in seventy how many times?
Five is contained in twenty how many times?
Five is contained in fifteen how many times?

LESSON SIXTH.

Six is contained in eighteen how many times?
Six is contained in twenty-four how many times?
Four is contained in twenty-four how many times?
Six is contained in thirty-six how many times?
Six is contained in forty-eight how many times?
Six is contained in fifty-four how many times?
Six is contained in seventy-two how many times?
Six is contained in eighty-four how many times?
Seven is contained in twenty-one how many times?
Seven is contained in twenty-eight how many times?
Five is contained in thirty-five how many times?
Seven is contained in forty-two how many times?
Seven is contained in fifty-six how many times?
Seven is contained in seventy how many times?
Seven is contained in sixty-three how many times?
Seven is contained in forty-nine how many times?
Seven is contained in eighty-four how many times?
Seven is contained in seventy-seven how many times?
Seven is contained in fourteen how many times?

LESSON SEVENTH.

Eight is contained in twenty-four how many times?
Eight is contained in thirty-two how many times?
Eight is contained in forty-eight how many times?
Eight is contained in sixty-four how many times?
Eight is contained in fifty-six how many times?
Eight is contained in seventy-two how many times?
Eight is contained in eighty how many times?
Eight is contained in ninety-six how many times?
Eight is contained in eighty-eight how many times?
Nine is contained in ninety how many times?
Nine is contained in fifty-four how many times?
Nine is contained in forty-five how many times?
Nine is contained in thirty-six how many times?
Nine is contained in twenty-seven how many times?
Nine is contained in seventy-two how many times?
Nine is contained in sixty-three how many times?
Nine is contained in eighty-one how many times?
Nine is contained in ninety-nine how many times?
Nine is contained in one hundred and eight how many times?

LESSON EIGHTH.

Ten is contained in twenty how many times?
Ten is contained in forty how many times?
Ten is contained in fifty how many times?
Ten is contained in sixty how many times?
Ten is contained in ninety how many times?
Ten is contained in seventy how many times?
Ten is contained in one hundred how many times?
Ten is contained in one hundred and ten how many times?
Ten is contained in one hundred and twenty how many times?

LESSON NINTH.

Eleven is contained in forty-four how many times?
Eleven is contained in thirty-three how many times?
Eleven is contained in fifty-five how many times?
Eleven is contained in seventy-seven how many times?
Eleven is contained in sixty-six how many times?
Eleven is contained in eighty-eight how many times?
Eleven is contained in ninety-nine how many times?
Eleven is contained in twenty-two how many times?
Eleven is contained in one hundred and twenty how many times?
Eleven is contained in 132 how many times?

LESSON TENTH.

Twelve is contained in thirty-six how many times?
Twelve is contained in sixty how many times?
Twelve is contained in eighty-four how many times?
Twelve is contained in ninety-six how many times?
Twelve is contained in 120 how many times?
Twelve is contained in 144 how many times?
Twelve is contained in 132 how many times?
Twelve is contained in 108 how many times?

QUEST. 1.—How many bushels of apples can be bought for one hundred cents, at twenty-five cents per bushel?

2.—How many barrels of flour may be bought for seventy dollars, at seven dollars a barrel?

3.—In how many hours will a man travel forty-eight miles, at the rate of eight miles an hour?

LESSON ELEVENTH.

QUEST. 1.—At six dollars a barrel, how many barrels of flour may be purchased for eighteen dollars? How many for twenty-four dollars? How many for thirty-six dollars?

If one barrel can be purchased for six dollars, for eighteen dollars as many barrels may be purchased as six is contained times in eighteen, which is three; therefore, for eighteen dollars, three barrels may be purchased.

2.—If raisins cost eight cents a pound, how many pounds may be bought for thirty-two cents? How many for forty-eight cents? How many for sixty-four? How many for seventy-two?

3.—If wood cost four dollars per cord, how many cords may be purchased for sixteen dollars? How many for twenty-four dollars? How many for twenty-eight dollars? How many for thirty-two dollars? How many for forty-eight dollars?

4.—How many bushels of apples may be bought for sixty shillings, at three shillings a bushel? How many for thirty shillings? How many for twenty-four? How many for eighteen?

5.—Eighty cords of wood are piled in eight different places; how many cords in each pile?

6.—A farmer sold wool to the amount of eighty-one shillings, at nine shillings a fleece; how many fleeces did he sell?

LESSON TWELFTH.

QUEST. 1.—How many thousand of boards may be bought for 144 dollars, at 12 dollars per thousand?

2.—How many books may be bought for eighty-four cents, *at seven cents* apiece?

3.—If thirty-six apples be divided equally between four boys, how many does each receive?

4.—How many barrels of flour may be bought for ninety-six dollars, at eight dollars per barrel?

Give the process.

5.—How many tons of hay may be bought for 120 dollars, at 12 dollars per ton?

6.—A man travels 144 miles in 12 days; how many miles does he travel in each day?

7.—A man purchased 11 acres of land for 132 dollars; how many dollars does he pay for each acre?

8.—One hundred and twenty-one dollars will pay for how many acres of land, at eleven dollars an acre?

9.—How many yards of cloth may be purchased for ninety cents, if one yard cost nine cents? How many yards for fifty-four cents? How many for forty-five?

LESSON THIRTEENTH.

QUEST. 1.—Paid twelve cents for a penknife; how many knives may be bought for thirty-six cents? How many for forty-eight? How many for sixty? How many for seventy-two? How many for ninety-six? How many for one hundred and eight?

2.—Purchased twelve quarts of plums, for eighty-four cents; what did I pay for each quart? What for three quarts? What for six quarts? What for seven quarts?

3.—A young man earned fifty-four dollars; by accident he lost four dollars; with the remainder he purchased land at 5 dollars per acre; how many acres did he purchase?

4.—Peter bought six oranges, at four cents apiece, and four lemons, at three cents apiece: he paid for them in labor, at nine cents per day; how many days did he labor?

5.—Purchased six barrels of sugar, at twelve dollars per barrel; paid for them in starch, at eight dollars per hundred pounds. How many hundred pounds did it take?

LESSON FOURTEENTH.

QUEST. 1.—A man bought sixteen bushels of wheat, at two dollars a bushel, and two barrels of flour, at eight dollars a barrel; paid for them in wood, at four dollars a cord. How many cords did it take?

What processes are required to solve this question?

2.—How many pounds of coffee, at eight cents per pound, will it take to pay for twelve pounds of sugar, at six cents per pound?

3.—A man bought sixteen pounds of raisins, at nine cents a pound, and paid for them in eggs, at twelve cents a dozen; how many dozens did it take?

4.—Bought ten cords of wood, at three dollars a cord; paid for it in hay, at five dollars a ton; how many tons did it take?

5.—A man travels 144 miles in 24 days; how many miles does he travel in one day? How many in four days? How many in five days? How many in twelve days?

LESSON FIFTEENTH.

QUEST. 1.—How many bushels of apples, at twenty cents a bushel, will it take to pay for four bushels of potatoes, at twenty-five cents a bushel?

What two processes are required to solve this question?

2.—Purchased groceries to the amount of one hundred dollars; paid for them in wood, at five dollars a cord. How many cords did it take?

3.—Purchased a horse for sixty-four dollars; paid sixty dollars in wood, at five dollars per cord, and the remainder in money. How many cords of wood did I pay? and how many *dollars in money*?

Give the process.

LESSON SIXTEENTH.

QUEST. 1.—A man bought two cows for twenty dollars each, three pigs for five dollars each, and a sheep for three dollars; how many dollars did he pay for the whole?

2.—Enoch had forty apples; he gave nine to one companion, and six to another; he then purchased ten more, and divided them between five of his schoolmates. How many did each receive?

3.—A man paid A. twenty dollars, B. ten, and C. ten, and divided the remainder of his money among his four sons, giving them five dollars apiece; how many dollars had he at first?

4.—From Concord to Nashua it is thirty miles; from Nashua to Lowell, ten miles; from Lowell to Boston, twenty miles. How many miles is it from Concord to Boston?

LESSON SEVENTEENTH.

QUEST. 1.—From New York to Newark it is ten miles; from Newark to Elizabethtown, five miles; from Elizabethtown to New Brunswick it is fifteen miles. How many miles is it from New York to New Brunswick?

2.—Two men travel from the same place; one travels five miles an hour, the other three; how far will they be apart at the end of eight hours?

3.—Two men travel from the same place in opposite directions; one travels at the rate of five miles an hour, the other at the rate of three; how many miles will they be apart at the end of eight hours?

4.—In a certain school ten study geography, fifteen study grammar, thirteen study arithmetic, and nineteen study little or nothing; how many scholars are there in the school? and how many more study than do not?

LESSON EIGHTEENTH.

QUEST. 1.—A man purchased five barrels of flour, at six dollars a barrel; he paid twenty-eight dollars for four barrels of fish. How many dollars did he pay for flour, and what did he pay a barrel for the fish? and how many dollars did he pay in all?

2.—If four men can do a piece of work in six days, in how many days can one man do it? In how many days can two men do it? In how many days can eight men?

3.—A family of eight persons have provision sufficient to last ten days; how many days would the same provision last sixteen persons? How many days would it last four persons?

4.—My grandfather is eighty years of age; my father is half as old, and I am half as old as my father: what is my age, and what is the age of my father? and how much older is my grandfather than I am?

LESSON NINETEENTH.

QUEST. 1.—I am now sixty years of age; fourteen years ago, I was just twice as old as my oldest son: what was his age then, and what is it now? and how many years older am I than my son?

2.—A pound of tea cost fifty cents, a pound of sugar ten cents, a pound of coffee eleven cents, and a pound of raisins sixteen cents: what cost the whole? and how much more cost the tea and sugar than the coffee and raisins?

3.—A man bought four yards of cloth, at four dollars a yard: three yards of satin, at three dollars a yard; five yards of silk, at three dollars a yard: he paid five ten-dollar bills. How many dollars must he receive back again?

4.—A man owed forty dollars; he paid at one time fifteen dollars; at another, fourteen; at another, seven: how many *dollars did he* pay in all, and how many remained due?

LESSON TWENTIETH.

QUEST. 1.—In one penny there are four farthings; how many farthings are there in four pence? How many in six pence? How many in eight pence? How many in ten pence?

2.—In one shilling there are twelve pence; how many pence are there in four shillings? How many in six shillings? How many in eight?

3.—How many farthings are there in one shilling? How many in two? How many in three?

4.—In one gallon there are four quarts; in one quart there are two pints. How many pints are there in two gallons? How many in four?

LESSON TWENTY-FIRST.

QUEST. 1.—In one pint there are four gills; how many gills are there in two quarts? How many gills in one gallon? How many in two?

2.—How many quarts are there in two gallons? How many pints? How many gills?

3.—Josiah bought seven lemons, at four cents apiece, and eight oranges, at six cents apiece; how many cents did he pay for the whole? and how much more did he pay for the oranges than for the lemons?

4.—There is an orchard consisting of twelve rows of trees, and nine trees in a row; how many trees are there in the orchard?

LESSON TWENTY-SECOND.

QUEST. 1.—A certain apple-tree bears four kinds of apples, and ten bushels of each kind; how many bushels does it bear? and what will they come to at ten cents a bushel?

2.—If five men can do a piece of work in three days, how many days would it take one man to do it? How many days would it take three men?

It is plain, that it would take one man five times as long as it would five men; and three, one third as long as it would one man.

3.—In twelve farthings, how many pence? How many in sixteen? How many in twenty? How many in thirty-two?

4.—If a man build forty rods of wall in eight days, how many rods will he build in one day? How many in three days? How many in five days?

5.—A man gave 144 dollars for twelve loads of hay; what did he pay for four loads?

6.—An orchard consists of one hundred apple-trees, an equal number in each row; how many are there in each row? How many are there in four rows?

LESSON TWENTY-THIRD.

QUEST. 1.—In two shillings and fourpence, how many pence? How many shillings in forty-eight pence? How many pence in forty-eight farthings?

2.—In a school of 96 scholars, an equal number study arithmetic, grammar, geography, philosophy, chemistry, and history. How many are there in each class?

3.—The parents of a family are each as old as the united age of all the children, six in number; the average age of the children is four more than their number. What is the age of the parents?

4.—A certain farm cost four hundred dollars, at twenty dollars per acre; how many acres are there?

5.—If two bushels of rye are equal to one bushel of wheat, and two bushels of oats are equal to one bushel of rye, how *many bushels* of oats are equal to twenty bushels of wheat?

SECTION V.

FRACTIONS.

If a unit, or a thing, be divided into parts, each of these parts is a fraction of the whole, and takes its name from the number of parts into which the thing is divided. A fraction, therefore, is a part of a thing. Two figures are necessary to express a fraction; they are written, one directly over the other. One half is written thus, $\frac{1}{2}$; one third thus, $\frac{1}{3}$. The figure below the line shows into how many parts the thing is divided. It is therefore called the denominator, or namer. The figure above the line shows the number of parts contained in the fraction. It is therefore called the numerator, or numberer. Thus, $\frac{1}{2}$; the 2 shows that the unit is divided into two parts; and the 1, that one of these parts is contained in the fraction. The numerator values the fraction; if, therefore, we increase it, the denominator remaining the same, we increase the value of the fraction. Thus, $\frac{1}{2}$, $\frac{2}{2}$; the value of the latter fraction is twice as great as that of the former, because the numerator is twice as large. If we increase the denominator, 2, the numerator remaining the same, we diminish the value of the fraction. Thus, $\frac{1}{2}$, $\frac{1}{4}$; the value of the latter is one-half the value of the former, because the denominator is twice as large. If the numerator of a fraction be equal to, or greater than the denominator, the fraction is called an improper fraction, because it is equal to a whole number. Thus, $\frac{2}{2}$ is an improper fraction. The unit is divided into two parts, and these two parts are contained in the fraction. A fraction and a whole number are sometimes written together, thus, $2\frac{1}{2}$. This expression is called a mixed number. A compound fraction is written thus, $\frac{1}{2}$ of $\frac{1}{4}$; it is a fraction of a fraction.

LESSON FIRST.

Teacher. If a thing, an apple, or a number, be divided into parts, by what general name would each of these parts be called?

Scholar. Fraction.

T. By what name would each part be called?

S. It would depend upon the number of parts into which the thing was divided.

T. What do you mean by that?

S. That the number of parts into which a thing is divided, gives name to each of those parts.

T. Illustrate your meaning.

S. If an apple be divided into two parts, or halves, each part is a half of the whole. If it be divided into three parts, each part is a third of the whole.

T. Can these halves be again divided into halves?

S. They can.

T. What part of the whole would one of this last division be?

S. One fourth.

T. How do you know?

S. The thing was, in the first place, divided into two parts, and then each of these parts was divided into two parts, and twice two are four; therefore, the whole would be divided into four parts, and one of these parts would be one-fourth of the whole.

LESSON SECOND.

Teacher. You have said that a thing may be divided into two parts, three parts, and four parts. Into how many parts may a thing be divided?

Scholar. Into as many as you please.

T. And how many of those parts would make a whole?

S. *As many* as the thing is divided into.

T. How do you know?

S. It is plain that all the parts into which a thing is divided, are equal to the thing itself.

T. What is the general name of the parts into which a thing is divided?

S. Fractions.

T. What then is a fraction?

S. It is a part of a thing.

T. James divided an apple into three parts, and gave away two of the parts; what part of the apple did he give away?

S. Two thirds.

T. How do you know?

S. If James divide his apple into three parts, these parts are called thirds; if he give away two of these parts, he gives away two thirds.

T. James had two apples, and gave away one of them; what part of what he had did he give away?

S. One half.

T. But a fraction is a part of a thing, and James gave away a whole apple. How will you reconcile this with your definition of a fraction?

S. Although he gave away a whole apple, yet as it was a part of what he had, it was a fraction.

LESSON THIRD.

T. If an apple cost three cents, what will one third of it cost?

S. One cent.

T. How do you know?

S. If an apple cost three cents, one third of it would cost one third of three cents; one third of three is one; therefore one third of the apple would cost one cent.

T. If three apples cost one cent, what will one apple cost?

S. One third of a cent.

T. Give the process.

S. One apple will cost one third as much as three apples;

if three apples cost one cent, one apple will cost one third of a cent.

T. How many halves in one whole?
How many wholes in three halves?
How many thirds in one whole?
How many thirds in three wholes?
Give the process.
How many wholes in six thirds?

S. If three thirds make one whole, in six thirds there would be as many wholes as three is contained times in six, which is twice; therefore in six thirds there would be two wholes.

T. How many fourths in one whole?
How many fourths in four wholes?
Give the process.
Two is one half of what number?
Two is one third of what number?
Two is one fourth of what number?
How do you know?

S. Four fourths make a whole, and four times two are eight; therefore eight is that number, of which two be one fourth.

T. How many twelfths in four wholes?
How many wholes in nine thirds?
How many wholes in eight fourths?
One half of two, is what part of three?
One third of three is one fourth of what number?
One fourth of four is what part of two?
One fourth of four is what part of three?
One fourth of eight is one half of what number?
One fifth of five is what part of ten?
Two fifths of five is how many fifths of ten?
Give the process.

LESSON FOURTH.

T. Three is what part of nine?
Three is what part of fifteen?

Three is one sixth of what number?
One sixth of eighteen is one fourth of what number?
One third of eighteen is one half of what number?
One third of eighteen is one fourth of what number?
Seven are how many times three?

S. Two and one third times.

T. Nine are how many times two?
Eleven are how many times three?
Thirteen are how many times four?
Seventeen are how many times five?

S. Three and two fifths times.

T. Seventeen are how many times three?
Nineteen are how many times four?
What part of seven is one?
How many sevenths are there in one whole?
How many sevenths are there in two wholes?
How many in three wholes? How many in five wholes?
How many in six? How many in seven?
What part of fourteen is seven?

S. One half.

T. Illustrate.

S. Seven is contained in fourteen twice; seven is therefore one half of fourteen, for twice seven is fourteen.

T. What part of sixteen is four?
What part of twenty is four?
Twenty are how many times four?
Twenty-five are how many times four?
Twenty-eight are how many times seven?
Thirty-two are how many times eight?

LESSON FIFTH.

Twenty-six is how many times 6? 7? 8? 9? 5?
Twenty-seven is how many times 6? 5? 9? 8?
Twenty-nine is how many times 7? 9? 8? 6? 10?
Twenty-three is how many times 5? 7? 8? 4? 9? 10?
Twenty-five is how many times 7? 9? 4? 6? 8? 5?

Twenty-three is how many times 9? 8? 6? 4? 5?
Thirty is how many times 9? 8? 6? 7? 5? 4?
Thirty-three is how many times 6? 5? 8? 9? 4? 7?
Thirty-four is how many times 8? 7? 9? 5? 6? 11?
Thirty-five is how many times 7? 8? 9? 6? 11? 5? 4?
Thirty-seven is how many times 6? 7? 8? 9? 12? 5? 4?

LESSON SIXTH.

Forty is how many times 8? 7? 6? 5? 9? 10? 11?
Forty-four is how many times 9? 11? 8? 7? 6? 5? 4?
Forty-five is how many times 11? 8? 9? 6? 7? 5? 4?
Forty-six is how many times 5? 6? 7? 8? 9? 10? 12?
Forty-nine is how many times 8? 7? 6? 9? 10? 11? 12?
Forty-eight is how many times 4? 7? 5? 6? 8? 9? 11?
Fifty is how many times 7? 8? 9? 11? 6? 12? 5?
Fifty-four is how many times 7? 9? 6? 8? 11? 12? 4?
Sixty is how many times 9? 8? 7? 6? 10? 5? 11?
Sixty-five is how many times 12? 8? 9? 5? 6? 7?
Sixty-six is how many times 7? 8? 6? 9? 5? 11? 12?
Seventy-two is how many times 10? 9? 8? 7? 6? 11?
Seventy-four is how many times 11? 8? 9? 7? 12? 6?
Seventy-six is how many times 8? 9? 7? 10? 11? 12?

LESSON SEVENTH.

QUEST. 1.—James bought two apples, and divided them between himself and two companions; what was the share of each?

Is one third of two apples the same as two thirds of one?

Teacher. Illustrate.

Scholar. In one apple there are three thirds; in two apples there are twice as many; twice three thirds are six thirds, and one third of six thirds is two thirds; therefore, one third of two is the same as two thirds of one.

2.—At four cents a yard, what will two and one half yards of tape cost?

Give the process.

3.—At three dollars a cord, what will two and one third cords of wood cost? What will two cords and two thirds cost?

Give the process.

4.—At four dollars a yard, what will three and three fourths yards of cloth cost?

Give the process.

5.—Twice four and one half of four are how many?

Twice four is eight, and one half of four is two, which, added to eight, makes ten.

6.—Twice three and one third of three are how many?

7.—Three times four and three fourths of four are how many?

8.—What will five and one half days' work come to, at two dollars a day?

Give the process.

LESSON EIGHTH.

QUEST. 1.—Five and one half times two are how many?

2.—Five and one half times three are how many?

3.—What will four and one half loads of bark cost, at three dollars a load?

4.—What will six tons of hay cost, at six and one half dollars a ton?

5.—Purchased six tons of hay for thirty-three dollars; what did the hay cost a ton?

Give the process.

6.—What cost twenty pounds of sugar, at five and one half cents a pound?

Give the process.

7.—Four and one half times three are how many?

8.—Six and one half times six are how many?

9.—Six times five and one half are how many?

10.—Twenty times five and one half are how many?

LESSON NINTH.

QUEST. 1.—*Teacher.* How many thirds are there in two wholes?

Scholar. Six.

T. How do you know?

S. In one whole there are three thirds, and in two wholes there are twice as many, and twice three are six; therefore, in two wholes there are six thirds.

2. How many fourths are there in 4? in 5? in 6? in 7?
3. How many thirds are there in 4? 5? 6? 8?
4. How many fourths are there in 4? 5? 6?
5. How many fifths are there in 2? 3? 4? 5?
6. How many sixths are there in 3? 4? 5? 6?
7. How many sevenths are there in 7 and 3 sevenths?
8. How many eighths are there in 8 and 2 eighths?
9. How many ninths are there in 9 and 2 ninths?
10. In three and three eighths, how many eighths?
11. In three and four eighths, how many eighths?
12. In 3 and one eighth, how many eighths?
13. In ten and one tenth, how many tenths?
14. In eleven and three elevenths, how many elevenths?

LESSON TENTH.

1. In sixteen fourths, how many times one? How do you know?
2. In ten fifths, now many times one?
3. In fifteen fifths, how many times one?
4. In eighteen sixths, how many times one? Give the process.
5. In fifty-one sevenths, how many times one?
6. In sixty-six eighths, how many times one?
7. In eighty-four ninths, how many times one?
8. In forty-five tenths, how many times one?
9. In thirty-five sevenths, how many times one?
10. In forty-five ninths, how many times one?
11. In fifty elevenths, how many times one?
12. In sixty-five ninths, how many times one?
13. In forty-five ninths, how many times one?
14. In fifty-five elevenths, how many times one?
15. In sixty tenths, how many times one?
16. In seventy-seven sevenths, how many times one?
17. In ninety-nine ninths, how many times one?
18. In eighty eighths, how many times one?

LESSON ELEVENTH.

QUEST. 1.—In four, how many fourths?

2.—In four, how many eighths?

3.—Which would you prefer, one fourth or one eighth par. of an apple?

S. One fourth.

T. For what reason?

S. Because it is larger.

T. How much larger?

S. Twice as large.

T. How do you know?

S. Because it takes twice as many eighths as fourths to make a whole.

4.—In four, how many fifths?

Which would you prefer, a fourth or fifth part of an apple?

For what reason?

How do you know?

5.—In four, how many thirds?

Which would you prefer, a fourth or a third part of an apple?

For what reason?

Illustrate.

6.—John had twenty-four cents; he gave one sixth of what he had to one boy, one eighth to another, and one fourth to another. How many cents did he give away? and what part of the whole had he left?

Give the process.

LESSON TWELFTH.

1. Three times 5 are how many times 4?

Process.—3 times 5 are 15, and 4 is contained in 15 three times and 3 fourths of a time, (not 3 times and $\frac{3}{4}$ of four.)

2. Three times 4 are how many times 5?
   Give the process.
3. Five times 4 are how many times 3?
   Give the process.
4. Three times 6 are how many times 5?
5. Three times 5 are how many times 6?
6. Five times 6 are how many times 3?
7. Four times 7 are how many times 8?
8. Four times 8 are how many times 7?
9. Five times 6 are how many times 7?
10. Five times 7 are how many times 8?
11. Six times 6 are how many times 9?
12. *Six times* 9 are how many times 6?

LESSON THIRTEENTH.

1. Three times 3 are how many times 2 and one half?

T. Give the process.

S. Three times 3 are 9; if in one whole there are two halves, in 9 wholes there are 9 times as many halves, and 9 times 2 are 18; in 9 wholes, therefore, there are 18 halves; in $2\frac{1}{2}$ wholes there are 5 halves, and 5 is contained in 18 three times and 3 fifths of a time.

T. But 5 halves are contained in 18 halves three times, and there are three halves remainder; are there not?

S. There are.

T. Why then do you say 3 times and $\frac{3}{5}$ of a time?

S. One whole is contained in one whole one time, and one half is contained in one half one time; therefore, 5 halves are contained in 18 halves the same number of times that 5 wholes would be in 18 wholes.

2. Five times 5 are how many times 3 and one third?

3. Four times 6 are how many times 4 and one half?

4. Six times 7 are how many times one half?

5. Seven and one half are contained in 25 how many times?

6. What part of 15 is 5?

How do you know?

7. Which is the greater fraction, one third, or 5 fifteenths?

How do you know?

If an apple be divided into 3 parts, and I give away one of these parts, I shall give away one third, for 3 times 1 are 3; if I divide an apple into 15 parts, and give away 5 of these parts, I shall give away one third, for 3 times 5 are fifteen. Therefore, ———.

8. Five and one third are contained in 24 how many times?

9. Which is the greater fraction, 8 sixteenths, or one half? Illustrate.

10. What part of 20 is 4?

11. What part of 25 is 5?

12. Which is the greater fraction, one fifth or one fourth?

LESSON FOURTEENTH.

OBSERVATION.—When the question is proposed, let the student give the answer, and then the process, not wait to be called upon for the process.

1. How many wholes are 5 times 2 fifths? Give the process.
2. How many are 6 times 3 sixths?
3. How many are 6 times 5 sixths?
4. How many are 7 times 3 fourths?
5. How many are 7 times 5 fourths?
6. How many are 8 times 4 sixths?
7. How many are 8 times 6 sixths?
8. How many are 9 times 5 eighths?
9. How many are 9 times 7 eighths?
10. How many are 10 times 6 sevenths?
11. How many are 10 times 7 sevenths?
12. How many are 11 times 7 eighths?
13. How many are 11 times 5 eighths?
14. How many are 7 times 8 elevenths?
15. How many are 7 times 9 elevenths?
16. How many are 6 times 7 tenths?
17. How many are 6 times 9 tenths?
18. How many are 5 times 8 ninths?
19. How many are 7 times 8 ninths?
20. How many are 4 times 6 eighths?
21. How many are 4 times 7 eighths?
22. How many are two fifths of five?
23. How many are 2 fifths of fifteen?
24. How many are 6 fifths of five?
25. How many are 6 fifths of ten?
26. How many are 3 sixths of six?
27. How many are 5 sixths of six?
28. How many are 8 sixths of six?
29. How many is 1 sixth of five?
30. How many are 6 times 1 sixth of 5?
31. How many are 5 times 1 sixth of 1?
32. How many are 5 times 1 sixth of 12?

33. One sixth of 12 is what part of 5?
34. One seventh of 14 is what part of 9?
35. One eighth of 8 is what part of 3?
36. Three fifths of 15 is how many times 3?

LESSON FIFTEENTH.

1. What is 1 fifth of 2?
2. What is 3 fifths of 2?
3. What is 2 fifths of 3?
4. What is 1 third of 5?
5. What is 6 thirds of 5?
6. What is 1 fifth of 7?
7. What is 10 fifths of 7?
8. What is 7 fifths of 10?
9. What is 4 sixths of 12?
10. What is two thirds of 12?
    What is the difference between 2 thirds and 4 sixths? Illustrate.
11. What is 3 fifths of 16?
12. What is 4 fifths of 17?
13. What is 5 sixths of 19?
14. What is 7 eighths of 40?
15. What is 6 sevenths of 35?
16. What is 5 sixths of 25?
17. What is 3 ninths of 40?
18. What is 7 ninths of 45?
19. What is 9 tenths of 55?
20. What is 5 ninths of 47?
21. What is 6 tenths of 55?
22. What is 3 tenths of 75?

LESSON SIXTEENTH.

QUEST. 1.—What will 8 bushels of wheat cost, at 2 and 4 sevenths dollars a bushel?

2.—A man builds 6 and 3 fourths rods of wall in one day; how many rods can he build in six days?

3.—What will 9 cords of wood cost, at 4 and 3 fourths dollars a cord?

4.—What will 7 and 3 fifths barrels of flour cost, at 6 dollars a barrel?

5.—A man can plant 1 acre and 1 third in one day; how many acres can he plant in 10 days?

6.—One acre of land yields 20 and 3 fourths bushels of corn; how many bushels would 8 acres yield?

7.—A man can hoe, in 10 days, 10 and 1 half acres; how many acres can he hoe in 1 day? (See Section I., on Fractions.)

8.—A man and boy hoe 3 acres in 3 days; the boy can hoe one half as much as the man. How much does the boy hoe in one day?

9.—A man can travel 3 and 1 half miles in one hour; how many miles can he travel in twelve and one half hours?

10.—How many are 12 times 3 and 1 half?

11.—How many are twelve and 1 half times 3?

LESSON SEVENTEENTH.

TABLE I. FEDERAL MONEY.

10 mills make	- - - -	1 cent.
10 cents "	- - - -	1 dime.
10 dimes "	- - - -	1 dollar.
10 dollars "	- - - -	1 eagle.

Repeat the Table of Federal Money.

QUEST. 1.—How many mills make 2 cents?

Give the process.

2.—How many mills make 10 cents? How many mills make 15 cents?

3.—How many mills make 2 dimes? How many mills make 3 dimes?

4. How many mills make 1 dollar?
5. How many mills make 1 eagle?
6. How many cents make 10 eagles?
7. How many cents make 10 dollars?
8. How many mills make 10 dollars?

TABLE II. STERLING MONEY.

4 farthings make	- - -	1 penny, *d.*
12 pence "	-	1 shilling, *s.*
20 shillings "	- -	1 pound, £
21, or 28 N. E. shillings	- -	1 guinea.

9. Repeat the Table of Sterling Money.
10. How many farthings in 2 pence?
11. How many farthings in 12 pence? Give the process.
12. How many pence in 2 shillings? Give the process.
13. How many farthings in 2 shillings? Give the process.
14. How many farthings in 20 shillings? Give the process.
15. How are pence reduced to farthings?
16. How are shillings reduced to pence?
17. How are shillings reduced to farthings?

OBS.—Denomination means kind, sort. Shillings, pence, and farthings are said to be of different denominations. Shillings are of a denomination higher than pence, and pence are higher than farthings; for a penny, or any number of pence, is of more value than a farthing or an equal number of farthings. Reducing higher denominations to lower, is called Reduction Descending. Reducing lower denominations to higher, is called Reduction Ascending.

18. How many shillings in 4 pounds?
19. How are pounds reduced to shillings?
20. Reducing pounds to shillings is what kind of reduction?
21. How, then, is Reduction Descending performed?

LESSON EIGHTEENTH.

QUEST. 1.—How many pence in 12 farthings?

If in 4 farthings there is 1 penny, in twelve farthings there would be as many pence as 4 is contained times in 12, which is 3; therefore, in 12 farthings there are 3 pence.

2. How many pence are there in 48 farthings?
Give the process.

3. Reducing farthings to pence is what kind of reduction?

4. How is Reduction Ascending performed?

5. How are farthings reduced to pence?

6. How many shillings in 24 pence?
Give the process.

TABLE III. TROY WEIGHT.

24 grains	make	1 pennyweight,	*dwt.*
20 pennyweights	"	1 ounce,	*oz.*
12 ounces	"	1 pound,	*lb.*

7. Repeat the Table of Troy Weight.

8. How many grains make 4 pennyweights?
Give the process.

9. Which is the higher denomination, pennyweights or grains?

10. How many pennyweights in 48 grains?

11. How are grains reduced to pennyweights?

12. How many ounces in 40 pennyweights?

13. How many pennyweights in 12 ounces?

TABLE IV. AVOIRDUPOIS WEIGHT.

16 drams, *dr.*	make	1 ounce,	*oz.*
16 ounces	"	1 pound,	*lb.*
28 pounds	"	1 quarter,	*qr.*
4 quarters	"	1 hundred weight,	*cwt.*
20 hundred weight		1 ton.	

14. Repeat the Table of Avoirdupois Weight.
15. How many pounds make 1 quarter?
16. How many drams make 16 ounces? Give the process.
17. How many quarters make 20 cwt.?
18. How many quarters in 56 pounds?
19. How many tons in 40 hundred weight? Give the process.

LESSON NINETEENTH.

TABLE V. CLOTH MEASURE.

2¼ inches	make	-	-	1 nail,	*na.*
4 nails	"	-	-	1 quarter,	*qr.*
4 quarters	"	-	-	1 yard,	*yd.*
3 quarters	"	-	-	1 ell Flemish.	
5 quarters	"	-	-	1 ell English.	
6 quarters	"	-	-	1 ell French.	

QUEST. 1.—Repeat the Table of Cloth Measure.

2. How many inches make 4 nails?
3. How many inches make 4 quarters? Give the process.
4. How many quarters make an ell English?
5. How many quarters make an all French?
6. How many yards in 32 nails? Give the process.
7. How many yards in 16 quarters? Give the process.

TABLE VI. WINE MEASURE.

4 gills	make	-	-	1 pint,	*pt.*
2 pints	"	-	-	1 quart,	*qt.*
4 quarts	"	-	-	1 gallon,	*gal.*

31½	gallons	make	-	1 barrel,	*bar.*
2	barrels	"	-	1 hogshead,	*hhd.*
2	hogsheads	"	-	1 pipe.	
2	pipes	"	-	1 tun.	

8. Repeat the Table of Wine Measure.
9. How many gills make 2 pints?
10. How many pints in 16 gills?
    Give the process.
11. How many gills in 4 quarts?
12. How many quarts in 32 pints?
    Give the process.
13. How many gallons in 2 pipes?
14. How many gallons in a hogshead?
15. How many hogsheads in a tun?
16. How many barrels in a tun?
17. How many tuns in 16 barrels?
18. To what kind of reduction does this question belong?

LESSON TWENTIETH.

TABLE VII. DRY MEASURE.

2	pints	make	-	1 quart,	*qt.*
8	quarts	"	-	1 peck,	*pk.*
4	pecks	"	-	1 bushel,	*bush.*

QUEST. 1.—Repeat the Table of Dry Measure.
2. How many pints in 8 quarts?
   Give the process.
3. How many pints in 4 pecks?
4. How many pecks in 16 pints?
   Give the process.
5. Which is the higher denomination, pecks or pints?
   How do you know?
6. *How many* bushels in 32 pints?
   Give the process.

TABLE VIII. MEASURE OF TIME.

60 seconds make - - -	1 minute,	*min.*
60 minutes " - - -	1 hour,	*h.*
24 hours " - - -	1 day,	*d.*
7 days " - - -	1 week,	*w.*
4 weeks " - - -	1 month,	*m.*
13 months, 1 day, and 6 hours, or 365 days and 6 hours - -	1 year,	*yr.*

For the explanation of the foregoing tables, see Part II.

7. Repeat the Table of Time Measure.

8. How many seconds in 60 minutes?

9. How many minutes in 24 hours?

10. How many hours in 120 minutes?
Give the process.

11. How many days in 48 hours?

12. How many weeks in 28 days?

13. To what kind of reduction does the foregoing question belong?
How do you know?

The common year consists of 365 days and 6 hours nearly. For the 6 hours, another day is added to February every 4 years, making 29 days in that month, and 366 in the year. This year is called Bissextile, or Leap Year. To know whether the year is common or leap year, divide it by 4, and if nothing remain, it is leap year; but if 1, 2, or 3 remain, it is first, second, or third year after leap year.

The year is divided into twelve months, called calendar months, because they are the months used in calendars or registers.

There is another division of the year, made by the revolution of the moon round the earth. This year, called the *lunar* year, consists of 13 months, called *lunar* months.

The year is also divided into four parts of three months each, called the four seasons.

The names of the months in each of the seasons are as follows:—

Spring	Summer.	Autumn.	Winter.
March,	June,	September,	December,
April,	July,	October,	January,
May.	August.	November.	February.

The order in which the months occur, and their length in days, may be seen by the following :—

Names.	Days.	Names.	Days.
1 January,	31	7 July,	31
2 February,	28	8 August,	31
3 March,	31	9 September,	30
4 April,	30	10 October,	31
5 May,	31	11 November,	30
6 June,	30	12 December,	31

14. How many days are there in the common year?

15. How many days in every fourth year?

16. What is this year called?

17. To which month is the day added?

18. How many days has February?

19. Into how many months is the common year divided?

20. What are these months called?

21. Why called calendar months?

22. How many lunar months are there?

23. By what is this division of the year made?

24. Repeat the months in each year, commencing with the spring.

25. Repeat the months of the year, commencing with January, and also the number of days in each month.

26. How many days in December?

27. How many days in February?

28. How many days in June?

29. How many days in August?

LESSON TWENTY-FIRST.

EXPLANATIONS AND DEFINITIONS.

1. The terms of a fraction are the numerator and denominator taken together.

2. A measure of two or more numbers is a number which will divide those numbers without a remainder.

3. The greatest common measure of two or more numbers is a number which will divide those numbers without a remainder.

4. A common multiple of two or more numbers is a number which may be divided by those numbers without a remainder. Hence the product of two or more numbers is their common multiple. 8 is a common multiple of 4 and 2, for twice four is 8.

5. The least common multiple of two or more numbers is the least number that can be divided by those numbers without a remainder. The least common multiple of 4 and 2 is 4.

6. A common multiple of the denominators of two or more fractions, is called their common denominator.

7. The common denominator of two or more fractions expresses the denomination to which each of the fractions must be reduced.

8. When any one denominator is a multiple of each of the others, it is their least common denominator.

9. A factor is a number employed as a multiplier.

10. A prime factor is a number divisible only by itself, or a unit, as 2, 3, 5, 7.

11. All the prime factors of a number are the least factors which, multiplied together, will produce it. 2, 2, and 3 are all the prime factors of 12.

12. Two numbers are prime to each other, when no one number will divide them both without a remainder. 8 and 9 are prime to each other.

13. A composite number is the product of two or more

numbers. 15 is a composite number; 3 times 5 are 15. 3 and 5 are called the component parts of 15.

14. A component factor of a number, is a composite number that will measure it. 4 and 6 are the component factors of 12; and 4, 6, and 12 are all the component factors of 12.

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1. What are the terms of a fraction?
2. What is a common measure of two or more numbers?
3. What is the greatest common measure of two or more numbers?
4. What is the least common multiple of two or more numbers?
5. What is a common multiple of the denominators of two or more fractions called?
6. How may it be found?
7. What does the common denominator of two or more fractions express?
8. What is a factor?
9. What is a prime factor?
   Give an example.
10. What are the prime factors of 8? of 16? of 18?
11. When are numbers prime to each other?
    Give an example.
12. What is a composite number?
    Give an example.
13. What are the component parts?
14. What is a component factor?

LESSON TWENTY-SECOND.

Let it be required to find a common denominator to the fractions $\frac{1}{2}$ and $\frac{1}{4}$, and to reduce the fraction to the denomination expressed by the common denominator.

Since 8 is a multiple of 2 and 4, we may reduce them both to eighths. But it is obvious to the eye, that 4 is also a *multiple of 2 and 4*, and that it is the least common multiple.

If we consider 8 as the common denominator, we must reduce both the fractions to eighths; if 4, then we have simply to reduce the $\frac{1}{2}$ to fourths.

To reduce a fraction of a higher denomination to a fraction of a lower, we first find a common denominator; then divide the common denominator by the denominator of each of the fractions; we thus ascertain how many of the lower denomination make one of the higher. Into the quotient thus obtained, we multiply the terms of each of the fractions, and the fractions are reduced to the denomination required. 8 contains 2, four times, therefore, four eighths make a half; and 8 contains 4 twice, therefore, 2 eighths make a fourth; 4 contains 2 twice—therefore, 2 fourths make a half. Their numerators added, give the answer.

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1. Reduce $\frac{1}{2}$ and $\frac{1}{4}$ to a common denominator, and add them together.
2. What is the common multiple of 2 and 6?
3. What is the least common multiple?
   How do you know?
4. How many twelfths make a half?
5. How many twelfths make a sixth?
6. How many sixths make a half?
   How do you know?
7. How many twelfths are $\frac{6}{12}$ and $\frac{3}{12}$?
8. How do you know that $\frac{6}{12}$ is equal to $\frac{1}{2}$?
   Because 6 is half of 12, and one is half of 2.
9. How do you know that $\frac{3}{12}$ is equal to $\frac{1}{4}$?
10. How many sixths are $\frac{3}{6}$ and $\frac{1}{6}$?
11. Reduce $\frac{4}{6}$ to thirds.
12. Reduce $\frac{8}{12}$ to thirds.
13. Which is the larger fraction, $\frac{4}{6}$ or $\frac{8}{12}$?
14. John had some nuts: he gave $\frac{1}{2}$ of what he had to his mother, $\frac{1}{3}$ to his sister, and $\frac{1}{6}$ to his brother; what part of what he had did he give away, and what part had he left?
15. James had some candy: he gave to one playmate $\frac{1}{3}$ of what he had; to another $\frac{1}{4}$; what part had he left?

In subtraction of fractions, we reduce them to a common denominator, as in addition, then subtract the less numerator from the greater.

16. A man sold $\frac{1}{3}$ of a ton of hay to neighbor A., $\frac{1}{5}$ of a ton to neighbor B.; what part of a ton had he left?

Give the process.

17. A man sold $\frac{1}{4}$ of a cord of wood to one man, $\frac{1}{2}$ to another; what part of a cord did he sell, and what part of a cord had he left?

Give the process.

18. How many shillings in 2 pounds?

19. How many twentieths in 2 wholes?

20. How many shillings in 2 pounds and 1 shilling?

21. How many 20ths in 2 wholes and $\frac{1}{20}$?

22. How many fourths in 1 whole and 1 fourth?

23. How many fourths in 1 half and 2 fourths?

24. Do you perceive any similarity between the addition of whole numbers of different denominations, and the addition of fractions of different denominators?

25. In what does the similarity consist?

LESSON TWENTY-THIRD.

1. $\frac{1}{2}$ of $\frac{1}{2}$ is how much?

2. What kind of a fraction is $\frac{1}{2}$ of $\frac{1}{2}$?

3. How is it reduced to a simple fraction?

4. What effect upon the value of a fraction has multiplying its denominator?

5. How does it diminish its value?

6. What effect has multiplying its numerator?

7. What effect has multiplying both numerator and denominator by the same quantity?

8. A man bestowed, in charity, a certain sum of money; to one object he gave $\frac{1}{3}$ of the whole sum, to another object $\frac{1}{4}$; what part of the whole sum had he then given away, and what part had he left?

9. *What is the sum* of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$?

10. From the sum of the foregoing fractions, subtract the sum of the following, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{16}$.

11. What is $\frac{1}{3}$ of $\frac{1}{2}$?

12. What is $\frac{1}{6}$ of $\frac{1}{8}$?

13. What is $\frac{1}{7}$ of $\frac{1}{9}$?

14. How do you obtain one ninth of a whole number?

15. How do you obtain one ninth of a fraction?

16. If the denominator of a fraction be 7, into how many parts is the unit divided?

17. If you multiply this denominator by 9, into how many parts is the unit now divided; and how much less is the value of each part than before?

18. How does multiplying the denominator divide the fraction?

19. If you multiply the numerator also by 9, will the value of the fraction be affected?

Illustrate.

20. In multiplying by a whole number, how many times do you repeat the multiplicand?

21. If your multiplier be 2, how many times is the multiplicand repeated?

22. How many times if the multiplier be 3? if 4?

23. How many times if the multiplier be 1?

24. How many times if the multiplier be $\frac{1}{2}$?

25. How do you obtain one half of a whole number?

26. Is one half of a fraction obtained by the same process?

27. How do you divide a fraction?

28. What is the quotient of $\frac{1}{6}$ divided by 2?

29. What is the product of $\frac{1}{6}$ multiplied by $\frac{1}{2}$?

30. What is the difference between the process of dividing a fraction by a whole number, and multiplying the same by a fraction?

31. If your multiplier be $\frac{1}{3}$, what part of the multiplicand will your product be?

32. How will you obtain it?

33. If your multiplicand be a fraction, how will you obtain $\frac{1}{3}$ of it?

34. How will you divide it by 3?

35. If your multiplier be $\frac{2}{3}$, what part of the multiplicand will your product be?

36. How will you obtain 2 thirds?

37. How do you multiply fractions by fractions?

38. Why do you multiply the denominator of the multiplier into the denominator of the multiplicand? and why the numerator of the multiplier into the numerator of the multiplicand?

LESSON TWENTY-FOURTH.

1. What kind of a number is $5\frac{3}{7}$?

2. How many sevenths in $5\frac{3}{7}$?

If in 1 whole there are 7 sevenths, in 5 wholes there would be 5 times 7 sevenths, which is 35 sevenths, and in 5 wholes and 2 sevenths there would be 37 sevenths; therefore, in $5\frac{3}{7}$ are there 37 sevenths.

3. How many fourths in $6\frac{3}{4}$?

Give the process.

4. What kind of a fraction is $\frac{27}{4}$?

5. How is a mixed number reduced to an improper fraction?

6. How many wholes in $\frac{27}{4}$?

7. How is an improper fraction reduced to a mixed number?

8. How many wholes in $\frac{27}{8}$?

How do you know?

9. Is the quotient of 27 divided by 8, 3 wholes and $\frac{3}{8}$ of a whole, or 3 wholes and $\frac{3}{8}$ of 8?

10. Is $\frac{3}{8}$ here employed to express a part of one unit, or of 8 units?

11. $\frac{3}{8}$ of 8 units would be how many units?

12. A man purchased 8 pounds of tea for $12\frac{3}{4}$ dollars; what did he pay a pound?

13. A man purchased tea for $\frac{51}{32}$ of a dollar a pound; what did he pay for 8 pounds?

14. How many times is 4 shillings contained in 1 pound? *Pounds must* be reduced to shillings.

15. How many times is $\frac{1}{8}$ contained in $\frac{1}{4}$?

Fourths must be reduced to eighths.

16. How many times is $2\frac{1}{3}$ contained in $8\frac{4}{5}$?

17. Is 11 fifths contained in 44 fifths the same number of times that 11 would be contained in 44?

18. What effect upon the quotient has dividing the divisor and dividend by the same quantity?

19. At $2\frac{3}{7}$ dollars a barrel, how many barrels of crackers can be bought for $4\frac{4}{7}$ dollars?

In the division of fractions, if the denominator of the divisor and the dividend be alike, divide the numerator of the dividend by the numerator of the divisor.

20. At $\frac{1}{7}$ of a dollar a bushel, how many bushels of apples can be bought for 56 dollars?

21. At $\frac{3}{4}$ of a dollar a bushel, how many bushels of pears may be bought for $\frac{3}{7}$ of a dollar?

The price of an individual article is always the divisor, and the amount of money to be expended is the dividend. The divisor, $\frac{3}{4}$, and the dividend, $\frac{3}{7}$, both, in their numerators, contain the factor 3. Now, to multiply or divide divisor and dividend by the same quantity, does not affect the quotient. The question may then read, How many bushels may be purchased for $\frac{1}{7}$ of a dollar, at $\frac{1}{4}$ of a dollar a bushel?

It is evident that at 1 fourth of a dollar a bushel, 4 times as many bushels may be purchased as at 1 dollar a bushel, and that for 1 seventh of a dollar, only 1 seventh as many bushels could be purchased as for 1 dollar. To multiply dividend is the same as to divide divisor; therefore, for $\frac{1}{7}$ of a dollar, at $\frac{1}{4}$ of a dollar a bushel, we could purchase 4 times 1 seventh, which is $\frac{4}{7}$ bushels, the answer.

In division of fractions by fractions, when the numerator of divisor and dividend are alike, *divide the denominator of the divisor by the denominator of the dividend.*

22. At $\frac{4}{5}$ of a dollar a pound, how many pounds of tea can be purchased for $\frac{4}{9}$ of a dollar?

Give the process.

23. What is the quotient of $\frac{4}{5}$ divided by $\frac{4}{9}$?

24. What is the quotient of $\frac{9}{7}$ divided by $\frac{9}{8}$?

25. At $\frac{4}{5}$ of a dollar a pound, how many pounds of sugar may be purchased for $\frac{7}{9}$ of a dollar?

In this example the divisor is divided by 8, and the dividend is divided by 9. But since to multiply dividend is the same as to divide divisor, and to multiply divisor is the same as to divide dividend, we may invert the divisor, and proceed as in multiplication. $\frac{8}{9}$ inverted, will be $\frac{9}{8}$; the question will then read, What is $\frac{4}{5}$ of $\frac{7}{9}$?

In all questions in division of fractions by fractions, we may invert the divisor, and proceed as in multiplication.

26. Divide $\frac{6}{7}$ by $\frac{1}{2}$, and illustrate.

27. What is the quotient of $\frac{3}{8}$ divided by $\frac{1}{2}$?

28. What is the quotient of $\frac{6}{7}$ divided by $\frac{4}{5}$?

29. What is the quotient of $\frac{9}{16}$ divided by $\frac{3}{4}$?

30. How do you divide one fraction by another, when their denominators are alike, and their numerators unlike?

31. How when their numerators are alike, and denominators unlike?

32. How when their numerators are unlike, and their denominators unlike?

If the terms of the dividend are divisible by the corresponding terms of the divisor, divide the terms of the dividend by the corresponding terms of the divisor. Thus, the quotient of $\frac{8}{16}$ divided by $\frac{2}{4}$, is $\frac{4}{4}$, equal to a unit, or 1.

LESSON TWENTY-FIFTH.

1. Divide $\frac{7}{8}$ by $\frac{4}{5}$.
2. Multiply $\frac{7}{8}$ by $\frac{4}{5}$.
3. From $\frac{7}{8}$ subtract $\frac{4}{5}$.
4. How do yo divide one fraction by another? Illustrate.
5. How do you multiply one fraction by another? *Illustrate.*

6. How do you subtract one fraction from another? Illustrate.

7. What is the difference between $\frac{1}{2}$ and $\frac{1}{6}$? Give the process.

8. What is the product of $\frac{1}{2}$ into $\frac{1}{6}$? Give the process.

9. What is the quotient of $\frac{1}{2}$ divided by $\frac{1}{6}$? Give the process.

10. A man purchased 9 yards of cloth for $27\frac{3}{4}$ dollars; what did he pay a yard?
Give the process.

11. A man bought 9 yards of cloth, at $3\frac{1}{12}$ dollars a yard; what did the whole cost?

12. A man sold $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{4}$ of his stock in a railroad; what part of his stock did he sell?

13. A man sold $\frac{1}{2}$ of $\frac{1}{3}$ of a house and lot, and retained the remainder, which he valued at 100 dollars; what was the value of what he sold?

14. A man purchased $\frac{1}{2}$ of $\frac{1}{3}$ of a threshing machine for 12 dollars; what was the whole machine worth?

15. A man distributed 6 apples among some boys, giving them $\frac{1}{2}$ of an apple each; how many boys were there?

16. A boy gave 12 apples to a number of his companions, giving $\frac{3}{4}$ of an apple to each; how many boys were there?

17. A boy gave to 9 companions $\frac{3}{4}$ of an apple apiece; how many apples did he give away?

18. How many more times will 9 contain $\frac{3}{4}$, than it will contain 3?

19. How many hours in $\frac{3}{4}$ of a day?

If in 1 day there are 24 hours, in one fourth of a day there would be $\frac{1}{4}$ of 24, which is 6; and if in one fourth there are 6 hours, in three fourths there would be 3 times 6, which is 18 hours.

20. How many minutes in $\frac{3}{4}$ of an hour?

21. How many shillings in $\frac{3}{4}$ of a pound?

22. How many farthings in $\frac{3}{4}$ of a pound?

LESSON TWENTY-SIXTH.

1. Into how many parts is a pound divided?
2. What part of a pound is 1 shilling?
3. What part of a shilling is a penny?
4. What part of a pound is a penny?
5. What part of a penny is a farthing?
6. What part of a shilling is a farthing?
7. What part of a pound is a farthing?
8. What part of a pound is 3 farthings?
9. What part of a pound is $\frac{3}{4}$ of a farthing?
10. What part of a dollar is 1 shilling?
11. What part of a dollar is 10 cents?
12. What part of a week is 4 days?
13. If a man earn 7 dollars in a week, how many dolls can he earn in 4 days?
14. If a man earn $\frac{3}{4}$ of a dollar in a day, how many dolls can he earn in 7 days?
15. If a man earn $\frac{4}{5}$ of a dollar in a day, how many dolls can he earn in 5 days? in 6? in 7?
16. In 7 shillings 6 pence, how many pence?
17. What part of a pound is 7 shillings 6 pence?
18. What part of 2 shillings 4 pence is 9 pence?
19. How many nails are there in 1 yard?
20. What part of a yard is 1 nail?
21. What part of a yard is 1 quarter and 2 nails?
22. A man pays 2 pence on a pound; what does he p on a debt of 18 pounds?
23. A man pays 2 pence on a shilling; what does he p on a debt of 4 pounds?
24. At 3 pence an ounce, what will 2 pounds cost?
25. In 64 nails, how many yards?

LESSON TWENTY-SEVENTH.

1. Reduce $\frac{8}{16}$ to its lowest terms.

A fraction is in its lowest terms, when no number greater than a unit will divide both numerator and denominator without a remainder. 8 will divide 8, and it will also divide 16; $\frac{1}{2}$ is therefore the lowest expression of the fraction $\frac{8}{16}$.

2. Reduce $\frac{3}{12}$, $\frac{4}{8}$, $\frac{6}{18}$, and $\frac{7}{21}$ to their lowest terms.

3. $\frac{6}{7}$ minus $\frac{1}{7}$ is how many times $\frac{1}{42}$?

4. $\frac{5}{12}$ minus $\frac{2}{7}$ is how many times $\frac{1}{21}$?

5. From the sum of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$, subtract the sum of $\frac{1}{3}$ and $\frac{1}{4}$, and divide the remainder by $\frac{1}{2}$.

6. What is the sum, product, and difference of $\frac{1}{2}$ and $\frac{1}{4}$?

7. How much is 6 times $3\frac{1}{2}$?

8. How many times is 6 contained in 21?

9. How many times is 6 contained in $3\frac{3}{7}$?

10. How many times is $\frac{1}{3}$ contained in $8\frac{1}{3}$?

11. How many times is $\frac{1}{4}$ contained in $\frac{4}{5}$?

12. Into how many parts may a thing be divided?

13. What gives name to these parts?

14. If your divisor be 2, what will the parts be called? If 3, what? If 4, what?

15. How many thirds make a whole?

16. How many fourths?

17. 1 is a half of what number?

18. 3 is a third of what number?

19. 5 is a fifth of what number?

20. 4 is a fourth of what number?

21. 4 is a fifth of what number?

22. 5 is a fourth of what number?

23. 35 is $\frac{5}{8}$ of what number?

If 35 is $\frac{5}{8}$ of some number, then $\frac{1}{5}$ of 35, which is 7, will be $\frac{1}{8}$ of that number; and if 7 is $\frac{1}{8}$ of that number, the whole will be 8 times 7, which is 56. Therefore, 56 is that number of which 35 is $\frac{5}{8}$.

24. 40 is $\frac{4}{5}$ of what number?
25. 5 is $\frac{5}{8}$ of what number?
26. 21 is $\frac{3}{7}$ of what number?
27. What number is $\frac{3}{7}$ of 49?
28. What number is $\frac{2}{5}$ of 25?
29. What number is $\frac{5}{6}$ of 30?
30. 4 is $\frac{2}{7}$ of what number?
31. $5\frac{1}{4}$ is $\frac{3}{7}$ of what number?
32. 9 and 3 fifths is 6 ninths of what number?
33. 17 is 4 times what number?
34. 19 is 3 times what number?
35. 20 is 8 times what number?
36. 21 is 9 times what number?
37. 7 is $4\frac{1}{2}$ times what number?
38. 11 is 1 fifth of how many times 5?
39. 15 is $\frac{3}{5}$ of how many times 5?
40. 25 is $\frac{5}{6}$ of how many times 15?
41. 40 is $\frac{5}{8}$ of how many times 16?
42. $\frac{5}{8}$ of 40 is what part of 50?
43. $\frac{5}{8}$ of 32 is $\frac{1}{2}$ of what number?
44. 45 is $\frac{3}{5}$ of how many times 5?
45. 35 is $\frac{5}{6}$ of how many times 10?
46. 36 is $\frac{6}{7}$ of how many times 21?
47. 27 is $\frac{3}{5}$ of how many times 10?
48. 18 is $\frac{6}{8}$ of how many times 6?
49. 28 is $\frac{4}{7}$ of how many times 7?
50. 56 is $\frac{7}{9}$ of how many times 12?
51. 64 is $\frac{8}{11}$ of how many times 8?
52. 72 is $\frac{6}{8}$ of how many times 16?
53. 84 is $\frac{7}{12}$ of how many times 12?
54. 96 is $\frac{8}{9}$ of how many times 9?
55. 6 and $\frac{2}{8}$ is $\frac{1}{8}$ of what number?
56. 7 and $\frac{1}{7}$ is $\frac{1}{14}$ of what number?
57. 9 and $\frac{1}{9}$ is $\frac{1}{3}$ of what number?

LESSON TWENTY-EIGHTH.

GENERAL EXERCISES.

1. One man owns $\frac{12}{16}$ of an acre of land; another owns $\frac{2}{8}$ of an acre. How many times as much land as the latter does the former own?

OBS.—In this example we may first make the denominators of the fractions alike; then divide the numerator of the dividend by the numerator of the divisor; or, we may divide the terms of the dividend by the corresponding terms of the divisor.

2. One man owns $\frac{5}{16}$ of an acre of land; another owns $\frac{2}{7}$ of an acre. How many times as much land as the latter does the former own?

OBS.—In this example the terms of the dividend cannot be divided by the corresponding terms of the divisor without a remainder. But to multiply the denominator is the same as to divide the numerator, and to multiply the numerator is the same as to divide the denominator. It is evident that $\frac{1}{7}$ of a unit would be contained in any number 7 times as many times as a unit, or 1; therefore $\frac{1}{7}$ would be contained in $\frac{5}{16}$ $\frac{35}{16}$ times, and $\frac{2}{7}$ would be contained in it one half as many times as $\frac{1}{7}$. Therefore, $\frac{2}{7}$ is contained in $\frac{5}{16}$ $\frac{35}{32}$ times. A fraction whose numerator is a unit is contained in any number as many times more than a unit as there are units in its denominator; therefore, multiply the numerator of the dividend by the denominator of the divisor. A fraction whose numerator is greater than a unit is also contained in any number as many times less than it would be were the numerator simply a unit as there are units in the numerator. Therefore, multiply the denominator of the dividend by the numerator of the divisor. This is the same as inverting the divisor. (See Written Arithmetic.)

3. Divide $\frac{3}{4}$ by $\frac{2}{3}$. How many times more is $\frac{1}{3}$ contained in any number than 1? What then do you do? How many times less is $\frac{2}{3}$ contained in any number than $\frac{1}{3}$? What then do you do?

4. A man sold a load of wood for $\frac{2}{16}$ of 64 dollars, and took his pay in oats, at $\frac{2}{3}$ of a dollar a bushel. How many bushels of oats did he receive?

5. A man owns $\frac{2}{3}$ of a house, valued at 300 dollars.

How many horses could he purchase with his share of the house; value of each horse $\frac{3}{4}$ of a hundred dollars?

6. What will $\frac{2}{3}$ of a hogshead of molasses cost, at $\frac{3}{7}$ of a dollar a gallon?

7. $\frac{1}{4}$ of a pound and $\frac{1}{4}$ of a penny are how many farthings?

8. $\frac{1}{8}$ of 240 farthings is what fraction of a pound?

9. Five men and three boys labor an equal number of days, and receive five pounds and four shillings. Each man receives four shillings a day, and each boy two shillings. How many days did they labor?

10. A man has 48 dollars to pay his laborers. He pays each man 6 shillings, and each boy 2 shillings; the number of men and boys was equal. How many were there of each?

LESSON TWENTY-NINTH.

1. A and B travel in opposite directions. A travels at the rate of $4\frac{1}{2}$ miles per hour; B travels at the rate of $3\frac{1}{2}$ miles per hour. How far will they be apart at the end of twelve hours?

2. C and D travel in the same direction; the one at the rate of $3\frac{1}{2}$ and the other at the rate of $5\frac{1}{2}$ miles per hour. How far will they be apart at the end of one hour and a half, and how far at the end of six hours?

3. Two men are 60 miles apart. They set out at the same time, and travel in the same direction. The foremost man travels at the rate of 3 miles an hour; the other, $3\frac{1}{2}$ miles an hour. In what time will he overtake the former?

4. One man can reap a piece of wheat in four days; another can reap the same in five days. In how many days would the two men reap the field, working together?

5. The third part of an army was killed, a fourth part was taken prisoners, and 1000 fled. How many were there in the army?

6. A, B, and C, would divide 100 dollars between them in such a manner that A may have 3 dollars more than B, and C 4 more than B. What is each man's share?

7. A man having a certain number of dollars, said, if the number were increased by its third, fourth, and sixth part, he should have 45 dollars. How many dollars had he?

8. A father, dying, left his fortune to his son, who spent $\frac{1}{8}$ of it the first six months. $\frac{2}{3}$ of the remainder lasted him four months longer, when he had left 40 dollars. What was the value of the estate?

LESSON THIRTIETH.

1. A merchant purchased hats for three dollars apiece, and sold them for four dollars apiece. What is the gain in laying out 24 dollars?

2. A man killed a third part of his flock of sheep; a fourth part died of disease, and the remainder, 100, he sold. How many sheep were there in the flock?

3. There is a rock which rises 40 feet above the surface of the water; $\frac{1}{2}$ and $\frac{1}{4}$ of it are beneath the surface. How high is the rock, and how deep is the water?

4. A man, driving his geese to market, was met by another man, who said, Good-morrow, Master, with your hundred geese. He replied, I have not a hundred; but, if I had as many more and half as many more as I now have, and two geese and a half, I should have a hundred. How many had he?

5. A hare has 40 rods the start of a greyhound, but

the greyhound runs 10 rods while the hare runs 3 rods. How many rods must the greyhound run to overtake the hare?

6. What number is that, which, being multiplied by $\frac{2}{3}$, the product will be 40?

7. What number is that, which, being multiplied by $\frac{1}{2}$, the product will be $\frac{1}{4}$?

8. A lad carried to market 8 quarts of raspberries and 6 quarts of blackberries. He sold his blackberries for 8 cts. a quart. He received in payment a jack-knife, valued at 25 cents, and the remainder in money. Not meeting with a ready sale for his raspberries, he exchanged them with a boy he met, for an equal number of quarts of blackberries, and gave his jack-knife into the bargain. He finally traded his blackberries for another knife, which he sold for $\frac{2}{3}$ of the price of the first knife. What did his knife cost him, allowing both kinds of berries to be worth 8 cents a quart?

LESSON THIRTY-FIRST.

1. One fifth of 25 is what fraction of 60?

2. One seventh of 84 is one twelfth of what number?

3. Three ninths of 27 is one ninth of what number?

4. Three sevenths of 21 is what fraction of 36?

5. Five twelfths of 60 is five times what number?

6. What number is that, which, being doubled and increased by 2, will be 28?

7. $\frac{7}{8}$ of 56 is seven times what number?

8. 121 is eleven more than ten times what number?

9. The price of meal in the month of June was one *dollar and eighty* cents a bag, two bushels in a bag. In *July the price* was one half more. In August the price

was three dollars a bag. What part of the first price was added the last month, and what was the price of the last per bushel?

10. Sold 20 bushels of corn at 90 cents a bushel, and with the money bought potatoes at 25 cents a bushel. How many bushels of potatoes did I buy?

11. 120 is ten times the double of what number?

LESSON THIRTY-SECOND.

1. A boy carried to market 12 quarts of gooseberries and 6 baskets of strawberries. Each basket contained 3 pints. He sold his gooseberries for 5 cents a quart, and his strawberries for 25 cents a basket. He received in payment raisins at 6 cents a pound, and beans at 8 cents a quart. The number of pounds of raisins was equal to the number of quarts of beans. What was that number, and what did he get a quart for his strawberries?

2. A boy went a fishing, and caught fish of different kinds and qualities, viz.: pickerel, perch, and eels. He met a gentleman, on his return, who offered him a certain price per fish, irrespective of kind or quality, and would take all he had. The boy would not accept the offer, but said he would take 12 cents apiece for his pickerel, 8 cents for his perch, and 4 cents for his eels. The gentleman took his fish, and paid him two dollars and forty cents,—the same in amount as first offered. What was the first offer?

On what subject did the boy show himself ignorant?

Of what particular rule?

3. A man owns a fruit-orchard, consisting of apple, pear, peach, and plum trees. $\frac{1}{4}$ are apple, $\frac{1}{5}$ pear, $\frac{1}{6}$ peach, and the remainder plum. What proportion *are plum?*

4. The relative value of the fruit produced on the several kind of trees (Ques. 3d) is proportioned to the number of trees of each kind. The value of all the fruit sold in one season was 36 dollars. What was the value of each kind?

5. A farmer owns a maple-orchard of 1000 trees. During the sugar season they yield, on an average, 4 quarts of sap a day, each tree. From an old tree 8 quarts will make a pound of sugar. But one half of the trees are young, and will make but half as much sugar from the same quantity of sap. How many pounds of sugar does the farmer make in one season, allowing 12 days to the season, and what is the value of the sugar at 8 cents a pound?

LESSON THIRTY-THIRD.

1. A man owns ten acres of land, lying in the form of a parallelogram. The length is 160 rods, and the breadth 10 rods. How many posts, one rod apart, will it take to surround it, and how many rails,—the fence to be five rails in height? And what will be the cost of the fence? The posts cost 20 cents and the rails 5 cents each.

OBS.—Teachers should exercise their classes frequently with questions similar to the following, using the terms *plus* and *minus* for addition and subtraction. Allow any member of the class to give the answer as soon as obtained.

2. *Teacher.* Add the following numbers in your mind as I repeat them, and give the result: 4, plus 2, plus 3, plus 5, minus 6, plus 7.

3. *T.* 6, plus 7, plus 8, minus 4, plus 3, minus 5.

4. *T. A boy* earned 50 cents, spent 6 for candy, 7 *for apples, 9 for gingerbread.* How many had he left?

5. *T.* A boy went a fishing and caught a variety of fish, and sold them all for one dollar. He expended one half of his money for a cap. One half of the remainder he gave to a poor boy. With one half of what he then had he purchased a pen-knife. What the boy now has is what fraction of what he received for his fish?

6. *T.* The milk-maid who carried milk to market in a pail upon her head contemplated purchasing eggs with the milk, which were to produce chickens, with which she was to purchase a dress. Suppose the following calculation to be made, how much did she really lose by spilling the milk, and how many quarts did she carry upon her head?

Milk worth 6 cents a quart.
Eggs " 1 cent apiece.
Allow one third to prove addled.
Chickens, 50 cents apiece.
Dress, 15 dollars.

7. *T.* A man caught in a net 7 dozen pigeons, minus 12, plus 6, minus 10, plus 4, minus 8, minus 12, plus 3, minus 9, plus 3, minus 7, plus 5. How many were there?

OBS.—The design of the foregoing and similar exercises is to wake up the class, give quickness and accuracy of thought. Hence the questions should be given off by the teacher in a spirited tone of voice, with a clear and distinct enunciation, and with such rapidity as to tax to the utmost the pupils' ability to follow.

LESSON THIRTY-FOURTH.

1. A and B started on foot for the same church, 8 miles distant. After walking 2 hours, A arrived just as the minister was commencing his discourse. At this *time B had made* $\frac{3}{4}$ of the distance to the church. Sup-

posing him to proceed at a uniform pace, how long would the sermon have been begun upon his arrival?

2. Two boys are contending with each other in driving the hoop. Upon the termination of the race, John's hoop had made 120 revolutions, and William's 140. John's hoop was 8 feet in circumference, and William's 6. Which arrived first at the goal, and how far had he driven?

3. John, Mary, and Jane, carry their dinner to school in the same basket. Jane is to have $\frac{2}{3}$ as much as Mary, and Mary $\frac{3}{4}$ as much as John. At noon they find their dinner consists of 9 cold sausages, 18 nut-cakes, and 18 apples. What was the share of each?

4. My mother's age is $\frac{3}{4}$ of my father's. Their united age is 84, which is just $1\frac{4}{3}$ of my own. What is the age of each of us?

5. When Jack got up and put on his clothes, he thought, if he could get to the wood, he should be quite well. But when he arrived there he was only half as well as he expected, but yet his health was twice as good as when he awoke. How sick was Jack at first?

6. A boy, on his way to school, met 3 poor children, and gave them all the money he had. He distributed it to them in the ratio of their ages. To the eldest he gave $\frac{1}{2}$, to the next in age $\frac{1}{3}$, and to the youngest he gave $\frac{1}{6}$. The youngest was 2 years old. What was the age of each of the other two?

7. One fourth of a guinea in New England shillings is $\frac{1}{7}$ of some number. $\frac{1}{12}$ of the same number, minus 1, is some fraction of a pound. What is that number and that fraction?

LESSON THIRTY-FIFTH.

GENERAL QUESTIONS.

1. How do you know that 2 and 3 are 5?
2. How do you know that 2 from 5 leaves 3?
3. How do you know that twice 3 are 6?
4. How do you know that 3 is contained in 6 twice?
5. What is the first of the foregoing processes called?
6. How is it proved?
7. What is the second called?
8. How is it proved?
9. What is the third process called?
10. How is it proved?
11. What is the fourth process called?
12. What is Addition?
13. What is Subtraction?
14. What is Multiplication?
15. Can Multiplication always be performed by Addition?
16. Can Addition always be performed by Multiplication?
17. When can it?
18. What is Division?
19. Can Division always be performed by Subtraction?
20. Can Subtraction always be performed by Division?
21. When can it?
22. What is a fraction?
23. How is a fraction produced?
24. Does division always result in a fraction?
25. When does it?
26. How many figures are necessary to express a fraction?
27. How are they written?
28. What is the figure below the line called?
29. Why so called?
30. What is the figure above the line called?

31. Why so called?

32. What is the value of a fraction?

33. When is a fraction in its lowest terms?

34. How reduced to its lowest terms?

35. What are the prime factors of 24? of 30? of 35?

36. What are the component factors of 20? of 30? of 40?

37. What is the difference between a prime factor and a component factor?

38. What is a common measure of 10 and 20?

39. What is the greatest common measure?

40. What is a common multiple of 8 and 16?

41. What is the least common multiple?

42. What is the difference between a common measure and a common multiple?

43. When may one of the denominators of a given number of fractions be considered their common denominator?

44. When no denominator is a multiple of each of the others, how may a common denominator be found?

45. In addition and subtraction of fractions, why is it necessary to find a common denominator?

46. How are a given number of fractions reduced to the same denomination?

47. How are fractions added?

48. How are fractions subtracted?

49. How are fractions multiplied?

50. How are fractions divided?

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